Working Paper

Non-Consecutive Executives

Changing Preferences in a Changing World

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Abstract

This paper proposes a formal model which provides intuition as to why voters would re-elect a politician who they previously removed from office. Given that voters once preferred an unknown challenger to the politician they removed, why would they return to this politician instead of trying yet another unknown challenger? I propose that politicians who stick to their guns and implement policy they care about rather than pandering to voters are more likely to be kicked out of office, but also the only type that voters will vote to return to office. The three period model involves a representative voter and up to three politicians. The dichotomous state of the world represents the issue most pressing to the voter. Politicians' ideal points are distributed uniformly between the possible states of the world. Without knowing for certain the state of the world, the politician must choose from among three policy options: two which match the possible state of the world and one in between. Voters want to match the current state of the world, which is unlikely to change. In the case the state of the world does change, voters will want to bring back a politician who previously matched the state of the world, as that politician would only have done so if she that policy was her primary choice. The model also allows for a politician to run for office and lose only to run again and win.

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1 Introduction

Why would voters choose to remove a politician from office only to re-elect the same politician at a later point? For this to be the case, voters would first decide they would prefer someone who had never been in office before to the person currently serving. This scenario is not at all difficult to imagine and in fact happens fairly regularly. The rubs arises in the fact that in order for the fired politician to return to office, the same voters, discontented with their subsequent choice, would need to decide that they would prefer their ex-representative to a new, untested challenger. If the voters preferred uncertainty to what they knew in the first election, why would they return to what they knew they did not want?

Winston Churchill provides possibly the most famous example of an executive serving non-consecutive terms. Although popular after successfully leading the United Kingdom through the Second World War, the British citizens did not believe he was the right candidate to lead the peace process. According to Benjamin Schneer,

> "[Britons] did not want to return to prewar conditions. They wanted a new Britain. Why else fight, and risk death to save it? They wanted government to guarantee health insurance, old-age insurance, family allowance, free education, decent housing and fully employment. But Winston Churchill had little sympathy with this outlook. Because he had limited interest in domestic policy, he failed to understand the power of the building wave of leftist sentiment in his county" (Schneer, 2015, p. xv).

His reputation as a war hero came as a mixed blessing; his opponents successfully used it to convince Britons that Churchill was not a peacetime hero (Addison, 2011). However in 1951, with Cold War tensions steadily increasing, Churchill was once again re-elected. Reconstructing Europe, the creation of the United Nations, and the advancement of the Iron Curtain all created an atmosphere in which domestic audiences recognized the importance of an internationally focused leader.

Churchill provides a notable example of an often overlooked phenomena. According to Goemans, Gleditsch and Chiozza's (2009) database Archigos, about 17% of world leaders since 1945 have served at least two non-consecutive terms. The instances of non-consecutive terms increase when looking at domestic, lower level office holders, such as mayors, governors, state legislators, etc. Despite the findings that career politicians care about their career as politicians, less work has looked at how this affects the behavior of politicians with a career-long time horizon. When office holding becomes a career, politicians become unwilling to surrender that career after a single loss. Rather than view each election as a individual game, these politicians are concerned about repeated interactions with their constituents. When examining the choices of a career politician, we need to examine a theoretically longer time horizon. A career time horizon is not necessarily the same as an election time horizon.

Current models of elections are ill-equipped to address the returning incumbent phenomenon for several reasons. Any two period model of electoral behavior immediately disqualifies itself, as one could not observe the first period politician returning to office because the game is over[2]. On the other hand, games with infinite

[2] This includes models of career politicians by Diermeier, Keane and Merlo (2005); Mattozzi and Merlo (2007), and Keane and Merlo (2010)

horizons typically discard candidates after they have been removed from office without allowing them the chance to return. Even if candidates *were* allowed to run for office again, the structure of existing games prevents any former office holders from being re-elected. Traditionally, elections serve as an incentive for politicians to adhere to the will of the voters (Barro, 1973; Ferejohn, 1986; Austen-Smith and Banks, 1989; Przeworski, Stokes and Manin, 1999; Adsera, Boix and Payne, 2003). Theorists have perturbed the structure of such models–changing the preferences of politicians, privatizing information, allowing for uncertainty regarding the state of the world–yet consistently, if the politician's actions diverge from the wishes of the voter, the voter removes the politician from office and that politician is never again considered a viable candidate.

Why would a politician enjoying the benefits of office then choose a policy knowing she is more likely to be removed from office than not? In order to return to office, a politician must first choose a policy that results in her removal from office. In order to understand a career politician, it is important to understand what drives her to run for office to begin with. Traditionally, the literature looks to policy and office benefits as the main motivations behind political careers (Downs, 1957; Wittman, 1983; Bernhardt and Ingerman, 1985; Callander, 2008). "Policy" has become a catch-all for passing legislation and non-pecuniary benefits that can only be acquired from political office. "Office benefits" refer broadly to monetary gains and, in the literature on career politicians, the advantages accrued in office that translate to higher salaries in the private sector. While politicians may care about both policies and the status or kickbacks from holding office, different politicians may weight these perks differently. Analysis finds career politicians stay in politics due to non-pecuniary rewards; that is, career politicians value the benefits associated with elected office more than monetary returns (Diermeier, Keane and Merlo, 2005; Mattozzi and Merlo, 2008; Keane and Merlo, 2010). Career politicians are more dedicated to implementing their policy preferences than "temporary" politicians — those who run for office then voluntarily leave for the private sector.

Even the most ideologically driven politicians, however, are not always able to pass their most ideal policy for a myriad of reasons; they are constrained not only by institutions but perhaps also their colleagues. These constraints are often overlooked in models of electoral behavior. Often the literature assumes the politician running for office can choose any policy she wishes once in office. In reality, this is often not the case. The scope of realistically available policy choices is limited by the need for some degree of consensus in decision making. In the model I present here, politicians exists on a continuum across the ideological spectrum, but are restricted to three possible policy choices. Restricting the set of policy choices available to the politician causes the weight of office value to vary by ideological position.

Examining re-election conditions calls to mind discussions surrounding the advantages and disadvantages of incumbency. The politician's prior experience appears as an advantage due to the fact the politician is indeed, eventually, re-elected. Incumbency advantage and its consequences, particularly in the United States' congress, have been demonstrated empirically (Erikson, 1971; King and Gelman, 1991; Levitt and Wolfram, 1997) and rationalized theoretically (Ashworth and Bueno de Mesquita, 2008; Ashworth, 2012). However, incumbency advantage does not account for the entirety of re-election as the incumbent looses her first chance at re-election, suggesting some element of incumbent *disadvantage*. Disadvantaged incumbents have also increasingly been the subject of scrutiny, particularly by scholars of poorer and less advanced democracies (Klašnja and Titiunik, 2013; Klašnja, 2015; Ariga, 2015). Neither incumbency advantage nor disadvantage can account for the whole story in this scenario, though. The discussions on incumbency advantage and disadvantage tend to occur independent of each other; political comebacks occur across all levels of development. Furthermore, studies of electoral advantage or disadvantage focus only on the election immediately following incumbency. A former elected official returning to office is not necessarily indicative of an incumbency advantage as the current incumbent faces defeat.

The decision of the politician is, of course, only one piece of the puzzle here. In order to restore a previously removed politician to office, the voter's preferences must change in such a way that although the voter preferred a random challenger to the politician at the end of the first period, by the end of the second period the voter now prefers re-electing the first period politician rather than a new random challenger. To accomplish this, I the tie voter's policy preferences to the state of the world and allow the state to change with some small probability.

For example, one could imagine a constituency deeply concerned about the economy. This constituency only wants a politician invests government resources in job creation and will punish a politician who diverts resources towards bolstering the military. Should a terrorist attack occur, the constituency would quickly change its mind. On the other hand, if the constituency is engaged in a military conflict, the citizens would prefer their politician to focus on military development, although a sudden economic crash might change their minds. Both a terrorist attack and economic crisis are unlikely events, but dramatically shift the preferences of the voter. If an especially hawkish politician or Wall Street type was removed from office in the first period and a terrorist attack or economic crisis occurs while they are out of office, the voter would be willing to bring her back.

The timing of the model in this paper vaguely parallels that found in pandering models. A key feature in pandering models is the voter's lack of information prior to decision making (Canes-Wrone, Herron and Shotts, 2001; Maskin and Tirole, 2004; Fox, 2007). In every pandering scenario, the politician is better informed than the voter and the voter will decide whether or not to re-elect the politician based on her uninformed beliefs. The asymmetry of information is necessary in order to induce pandering; however, reinstatement of a politician can never happen under the construction of pandering models. Since the voters do not learn for certain their true preference until the game is over, they are never able to meaningfully update their expectations in such a way that would make them want to bring back a politician they already threw out. The voter must make decisions while still uninformed.

In this model, the information asymmetry leans the other way. While the voter does not know for certain the state of the world in the next period, similar to pandering models, she does know the state of the world in the current period after the current period's policy has been announced. The voter is more informed than the politician when the politician chooses a policy. This allows for the voter's preferences to shift in meaningful ways and potentially allow the return of a previous politician.

The model presented here bears some resemble to the model described in Duggan (2000). The voters (or single voter in this case) are unsure of a single politician's ideal point, but are informed about the distribution from which the politician is drawn. The resulting behavior of politicians is also very similar. Politicians with extreme policy preferences (those that match a possible state of the world) never compromise while politicians with more moderate ideal points tend to be much more flexible with their policy choice, catering to the likely desire of the voter. Furthermore, in both equilibria the equilibrium play policy choices depend only on the politician's ideal points and voters use prospective voting.

Differences between the models arise from having a single voter who only cares about the extreme policy choices. In Duggan's model, the median voter is the expectation of some distribution of the electorate, thus politicians who are willing to do so compromise inward, resulting in policy symmetry. In this model the voter prefers the politician to pick an *extreme* policy causing asymmetric policy choices in the second period.

The rest of the paper is organized as follows. Section 2 describes the model in detail. Section 3 presents the results, highlighting a few key aspects and comparative statics of the equilibrium. Finally, section 4 briefly describes extensions of the model and concludes.

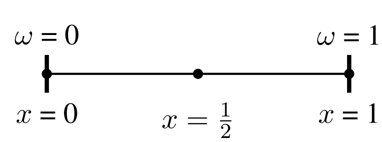
2 The Model

There exists two possible states of the world $\omega \in \{0,1\}$. The state of the world changes with some probability ρ. The states of the world can be thought of as the issue most important to voters; i.e. the economy versus military spending. Formally I will not address why the world changes, just that it does. Substantively, the change can be brought about by changes to the voters condition that are independent of politics. For example, a terrorist attack could shift the voters' priority from job creation to military enhancement. Likewise, a stock market crash may cause voters to value an economically trained politician rather than one with a military background. Luckily for most countries, neither terrorists attacks nor stock market crashes are very common. Once the voters' preferences have shifts, they are likely to remain focused for some time.

Just as the state of the world is drawn from a discrete set, the available policy options are also discretized. There are three possible policy choices, $x_t \in \{0, \frac{1}{2}, 1\}$. Two policies match the possible states of the world and one is a "moderate" policy between the two. The policies which match the state of the world can be thought of as allocating all resources to that state, i.e. economy or military spending. The median policy can be thought of as dividing resources between the two.

The game lasts for three periods and there is no discounting.

Figure 1: *Here is a graphical representation of the state of the world (vertical lines), available policy choices (circles), and politician types (horizontal line). The politician types will be described in detail in section 2.1.*



2.1 The Politicians

There are up to three politicians, i, who each have an ideal point $\hat{x}_i \in$ [0,1]. Ideal points are distributed uniformly on the interval [0,1]. A politician in office may choose a policy $x_t \in \{0, \frac{1}{2}, 1\}$. Although there are only two possible state of the world, the ideal points of politicians exist in a continuum between the possible states of the world. In substantive terms keeping with the economic policy versus military spending analogy, each politician has some preference for dividing her attention between economic and military policy. Depending on where the politician's ideal point lies in relation to the available policy choices, the politician has strict preference ordering over the policy options. When the politician believes the state of the world is far from her first preference policy, she must decided whether or not the benefit of being in office is worth compromising on policy in order to secure immediate re-election.

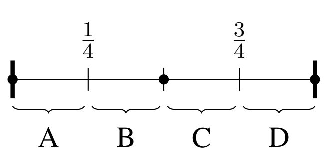
A politician with ideal points $\hat{x}_i$ has a utility function:

$$u_i(x|\hat{x}_i) = \begin{cases} -|x_t - \hat{x}_i| + B & \text{if in office} \\ 0 & \text{if not in office} \end{cases} \tag{1}$$

where $B \in (\frac{1}{4}, \frac{1}{2})$ is a benefit the politician receives from being in office. Limiting the office benefit to this range ensures a politician will compromise to at most one other policy position[3]. Substantively, this wards against politicians who are willing to choose any policy so long as they can stay in office. Career politicians care about policy.

Let x' be the policy choice that is closest to a politician's ideal point. Let x'' be the second closest policy choice to a politician's ideal point. Finally, let x''' be the policy choice that is farthest from a politician's ideal point. With these definitions, I can divide the continuum of politician types into four major groups which are depicted in Figure 2.

Figure 2: *This figure displays the four group intervals a politician may fall in: A, B, C, or D. The thick lines at either end of the interval represent the possible states of the world and the circles represent the policy choices.*



Each group has its own strict preference ordering. For politicians in group A: $x' = 0$, $x'' = \frac{1}{2}$, and $x''' = 1$. For politicians in group B: $x' = \frac{1}{2}$, $x'' = 0$, and $x''' = 1$. For politicians in group C: $x' = \frac{1}{2}$, $x'' = 1$, and $x''' = 0$. For politicians in group D: $x' = 1$, $x'' = \frac{1}{2}$, and $x''' = 0$. Table 1 provides the preference ordering for all politicians in each of the four groups.

[3]See full equilibrium proof in the appendix.

Table 1: *Preference Orderings by Group*

Group	x'	x''	x'''
A	0	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	1
C	$\frac{1}{2}$	1	0
D	1	$\frac{1}{2}$	0

In every period t, the in-office politician chooses a policy. Let α_t be the cutoff such that every type to the left of α_t chooses $x_t = 0$. Let β_t be the cutoff such that every type to the right of β_t chooses $x_t = 1$. Then, every politician with an ideal point in the range (α_t, β_t) will choose $x_t = \frac{1}{2}$.

In the second period, the cutoffs will depend on the revealed state of the world in the first period and which politician is in office. Let α_{2,ω_1} and β_{2,ω_1} be the cutoffs for a first period incumbent elected to the second period. Let α'_{2,ω_1} and β'_{2,ω_1} be the cutoffs for a random challenger elected to the second period.

The state of the world is not revealed until *after* the politician in office chooses a policy. Let the politician's belief about the current state of the world given the previous state of the world be denoted $\mu_{\omega_t}(\omega_{t-1})$. The voter has these beliefs in common with the politician. Though the voter learns the current state of the world after the politician makes a policy choice, the voter does not know for certain what the state of the world will be in the next period. Each state of the world is perceived to be equally likely prior to the first period politician choosing a policy. Based on the politician's type and belief about the state of the world, the politician will choose some strategy $\sigma^i(\hat{x}_i, \omega_t, \mu_\omega) = x_t$.

2.2 The Voter

This model assumes a single representative voter. If the voter's primary concern is the economy, the voter prefers the politician in office to choose a policy which appropriately address issues relating to the economy and not choose a policy which increases military spending. The voter only cares about the chosen policy matching the state of the world and thus the voter's utility is as follows:

$$u_v(x_t|\omega_t) = -|\omega_1 - x_t| \tag{2}$$

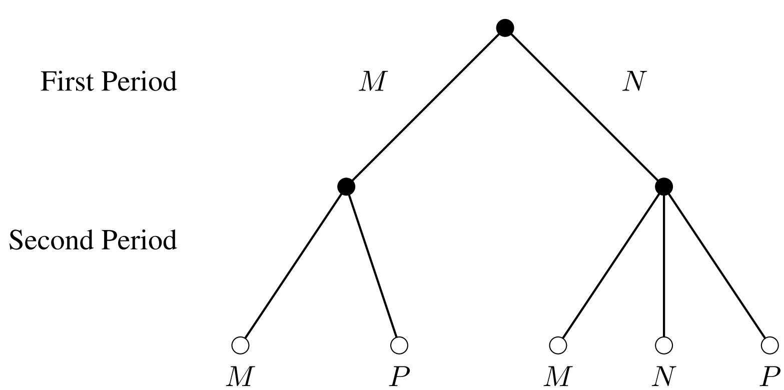
The voter may choose to re-elect the current politician in office or not. If the politician in office is not re-elected, the voter may choose a randomly drawn challenger, or, if applicable, a politician who was previously removed from office.

Though an individual politician's ideal point is private information, the distribution of ideal points is common knowledge as is the policy choice of the politician in office; therefore, the voter has a belief about the ideal point of a politician who has made a policy choice. Many politicians with distinct ideal points adopt choose the same policy, so voter's beliefs are distributed across some range. The most basic notation for the voter's belief is $\mu^v_{\hat{x}_i}(x_t)$, but the belief will also be affected by the state of the world in the previous period and the previous actions taken by the politician, if applicable.

The voter's strategy is a function of the voter's belief over the politician's type and the state of the world, denoted $\sigma^v(x_t|\mu_g, \omega_t) = i$. The notation regarding the voter's choice of politician is a bit confusing. Ultimately, there may be up to three politicians in office. Let the politician the voter chooses come from a set of these politicians; i.e., $i \in \{M, N, P\}$. Specifically, let politician M be in office during the first period, let politician N be the random challenger competing against politician M at the end of the first period, and politician P be the random challenger running for office at the end of the second period. I do not assume anything about the preferences or behavior of politicians $\{M, N, P\}$ other than the general assumptions made about all politicians detailed in the previous section. I simply label them as such for clarity.

Figure 3 illustrates the possible decision paths of the voter. In the first period, the voter has only two options. Depending on the choice the voter made in the first period, she may be able to choose between two or three possible candidates. The second branch of the tree, in which the voter elects the random challenger, is the focus of this paper. Traditional models of elections do not offer the previous incumbent an opportunity to run again. It should be noted, though, the first branch of the tree, in which the incumbent is re-elect to the second period, still illustrates interesting political behavior. Since the voter learns nothing about a politician who was not elected, it is possible that the random challenger at the end of period one is the same politician as the random challenger running at the end of two. It is possible for a politician to loose her first bid for office, then run again and win. This behavior is also not witnessed in traditional models of elections.

Figure 3: *This is the decision tree for the voter.*



2.3 Timing

The timing of each period in the model is as follows. While the state of the world may change at the beginning of each period, the probability is sufficiently small.

1. Nature chooses a state of the world.
2. The politician in office chooses a policy.
3. The state of the world is revealed.
4. The voter decides which politician holds office in the next period.

In the final period, there is no choice for the voter to make as the game ends.

3 Results

3.1 Equilibrium

I begin by presenting a fully characterized equilibrium[4]. The nature of the model (three periods, changing states, etc.) results in a fairly extensive, notation-heavy equilibrium. As such, I will provide notation and intuition for each period separately. The strategies follow a general pattern, as all policies are chosen according to cut-points primarily determined by the office benefit and the state of the world. The voter forms beliefs about the politician's type knowing policy is decided along these cut-points. All on-path beliefs are updated using Bayes' rule. Formal notation describing equilibrium in its entirety inclusive of both on- and off-path beliefs is provided in the appendix.

3.1.1 First Period

In the first period, only the incumbent, M, and the voter make choices.

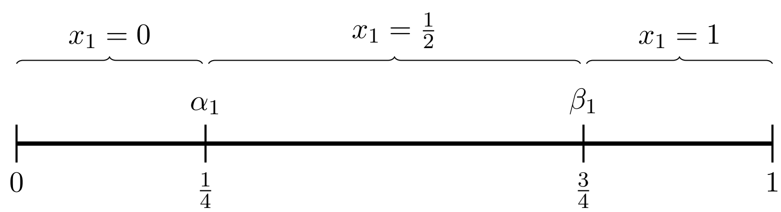
$$\sigma_1^{*M} = \begin{cases} x_1 = 0 & \text{if } \hat{x}_M \in [0, \alpha_1] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_M \in (\alpha_1, \beta_1) \\ x_1 = 1 & \text{if } \hat{x}_M \in [\beta_1, 1] \end{cases} \qquad \sigma_1^{*v} = \begin{cases} M & \text{if } x_1 = \omega_1 \\ M & \text{if } x_1 = \frac{1}{2} \\ N & \text{if } x_1 = -\omega_1 \end{cases}$$

$$\alpha_1 = \frac{1-B}{3} \quad \beta_1 = \frac{2+B}{3}$$

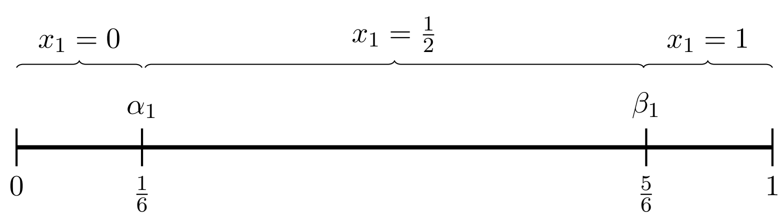
[4] I conjecture this is the unique equilibrium.

The incumbent believes each state of the world is equally likely. Politicians of different types choose a policy based on some cut-point determined by the size of the office benefit. The voter's belief about the incumbent's type is uniformly distributed across three intervals defined by the cut-points along which politicians choose policy. The behavior of the voter is discussed in Proposition 2. Figure 4 illustrates the behavior of politicians in the first period. The exact cutoffs for incumbent behavior depend on the size of the office benefit, B. Figures 4a and 4b show the cutoffs for the politician's action choice at when the office benefit, B, is at its minimum and maximum, respectively. Since the state of the world will not be revealed until after a policy choice is made, the policy choice does not depend on the probability the state of the world will change.

Figure 4: *First Period Behaviors: Politicians with ideal points from* 0 *to* α_1 *choose* $x_1 = 0$*; politicians with ideal points from* α_1 *to* β_1 *choose* $x_1 = \frac{1}{2}$*; and politicians with ideal points from* β_1 *to* 1 *choose* $x_1 = 1$*. Exactly where* α_1 *and* β_1 *depends on the size of the office benefit* B*. Figure 4a shows the cutoffs for policy decisions when the office benefit is that the minimum and Figure 4b picture shows the cutoffs for policy decisions when the office benefit is at the maximum. Note that when the size of the office benefit increases, more politicians hedge their bets by choosing the relatively safe policy of* $x = \frac{1}{2}$.



(a) *Cutoffs when the office benefit is at its minimum; i.e.,* B *arbitrarily close to* $\frac{1}{4}$



(b) *Cutoffs at the maximum office benefit; i.e.,* B *arbitrarily close to* $\frac{1}{2}$.

3.1.2 Second Period

There are two possible scenarios in the second period, each of which will be discussed individually. First, it could be that the first period incumbent M was re-elected to office for a second term. Second, and the general focus of this paper, describes what happens if the first period incumbent M lost the race for re-election and the random challenger became the incumbent for the second period, N.

When M is in office:

$$\sigma_2^{*M}(\omega_1) = \begin{cases} x_1 = 0 & \text{if } \hat{x}_M \in [0, \alpha_{2,\omega_1}] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_M \in (\alpha_{2,\omega_1}, \beta_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_M \in [\beta_{2,\omega_1}, 1] \end{cases} \qquad \sigma_2^{*v}(M, \omega_2) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases}$$

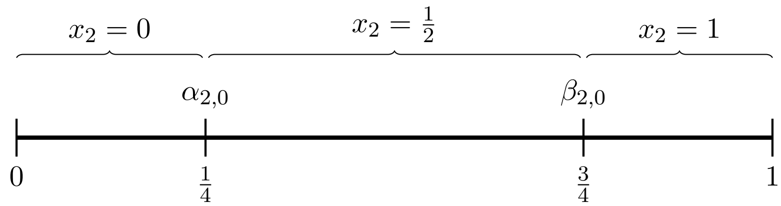
$$\alpha_{2,0} = \frac{2(1-\rho)B + \rho}{2 + 2\rho} \quad \beta_{2,0} = \frac{3 - 2\rho B - \rho}{2(2-\rho)}$$

$$\alpha_{2,1} = 1 - \alpha_{2,1} \quad \beta_{2,1} = 1 - \beta_{2,0}$$

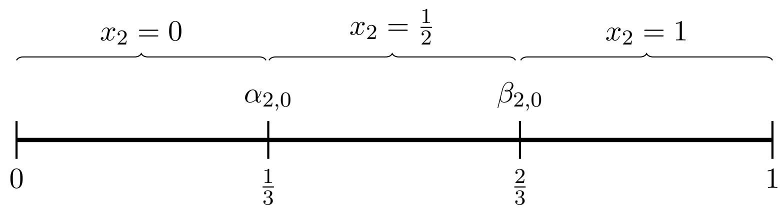
The second period strategy of the first period incumbent, σ_2^{*M}, is a cut-point strategy, just as in period one. The cut-points are now not only based on the office benefit, but also what the state of the world was revealed to be during the first period. The voter will only re-elect the politician if she chooses the policy which matches the what state of the world is revealed to be in the second period. Figure 5 illustrates the actions taken by the politician in the second period. Between Figures 5a and 5b, the value of the office benefit varies while keeping the probability the state of the world changes close to zero.

Figure 5: *Re-elected incumbent actions in the second period when the state of the world in the first period was revealed to be zero. If the incumbent's ideal point falls between 0 and $\alpha_{2,0}$, she will choose $x_2 = 0$. If her ideal point is between $\alpha_{2,0}$ and $\beta_{2,0}$, she will choose $x_2 = \frac{1}{2}$. Finally if her ideal point is between $\beta_{2,0}$ and 1, she will choose $x_2 = 1$. Exactly where α_2 and β_2 are dependent on the size of the office benefit B and the probability the state of the world will change, ρ. Figure 5a and 5b show the cutoffs for incumbent behavior when B is at its minimum and maximum, respectively, and ρ is close to zero.*

(a) *Cutoffs when the office benefit is at its minimum and the probability the state of the world will change is close to zero; i.e., $B = \frac{1}{4}$ and $\rho = \epsilon > 0$.*



(b) *Cuffos when the office benefit is at its maximum and the probability the state of the world will change is close to zero; i.e. $B = \frac{1}{2}$ and $\rho = \epsilon > 0$.*



When N is in office:

$$\sigma_2^{*N}(\omega_1)\begin{cases} x_2 = 0 & \text{if } \hat{x}_N \in [0, \alpha'_{2,\omega_1}] \\ x_2 = \frac{1}{2} & \text{if } \hat{x}_N \in (\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_N \in [\beta'_{2,\omega_1}, 1] \end{cases}$$

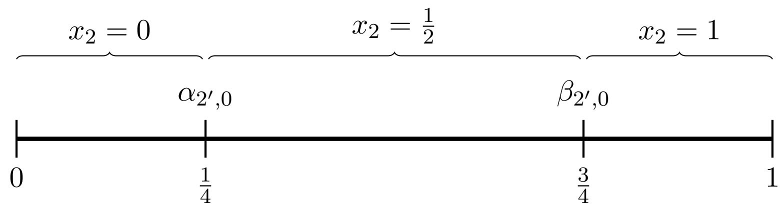
$$\sigma_2^{*v}(N, \omega_2 = \omega_1) = \begin{cases} N & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(N, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ M & \text{if } x_2 = \frac{1}{2} \\ M & \text{if } x_2 = -\omega_2 \end{cases}$$

$$\alpha'_{2,0} = \frac{2(1-\rho)B + \rho}{2 + 2\rho} \qquad \beta'_{2,0} = \frac{3}{4}$$
$$\alpha'_{2,1} = 1 - \beta'_{2,0} \qquad \beta'_{2,1} = 1 - \alpha'_{2,0}$$

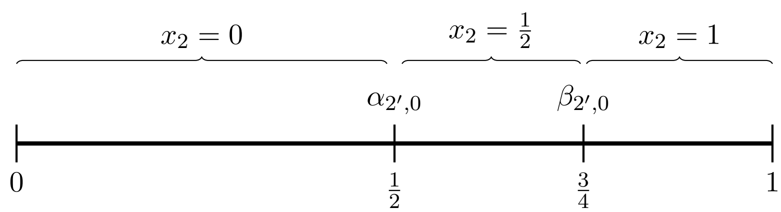
Alternatively, the first period incumbent could be removed from office at the end of the first period and replaced by some random challenger. In this case, the random challenger becomes the second period incumbent, N. The second period incumbent, just as any politician in office during any period, plays a cut-point strategy. If the state of the world remains the same as it was in the first period, the voter adopts the same strategy as she would play if the first period incumbent had been re-elected. That is, if the politician matches the state of the world, the voter will re-elect her. Otherwise, the voter will opt for a random challenger. If the state of the world changes, however, the voter reinstate the first period incumbent, M, regardless of the policy choice made by the current politician, N. This result is is detailed in Proposition 3 and discussed at greater length then. Figure 6 illustrates the behavior of the second period incumbent. The value of the office benefit is varied between Figures 6a and 6b while holding the probability the state of the world will change close to zero.

Figure 6: *Random Challenger actions in the second period when the state of the world was revealed to be zero. If the Random Challenger's ideal point falls between* 0 *and* $\alpha_{2',0}$ *she will chose* $x_2 = 0$ *as her second period policy; if her ideal point falls between* $\beta_{2',0}$ *and* 1*, she will choose* $x_2 = 1$ *as her second period policy; and finally, if her ideal point falls between* $\alpha_{2',0}$ *and* $\beta_{2',0}$*, she will choose* $x_2 = \frac{1}{2}$ *as her second period policy. In equilibrium,* $\alpha_{2',0}$ *depends on the office benefit,* B *and the probability the state of the world will change,* ρ*. Figure 6a and 6b show the cutoffs for the Random Challenger's behavior when the office benefit,* B*, is at its minimum and maximum, respectively, and the probability the state of the world will change,* ρ *is close to zero.*

(a) *Cutoffs when the office benefit is at its minimum and the probability the state of the world will change is close to zero; i.e.,* B *arbitrarily close to* $\frac{1}{4}$ *and* $\rho = \epsilon > 0$*.*



(b) *Cutoffs when the office benefit is at its maximum and the probability the state of the world will change is close to zero; i.e.,* B *arbitrarily close to* $\frac{1}{2}$ *and* $\rho = \epsilon > 0$*.*

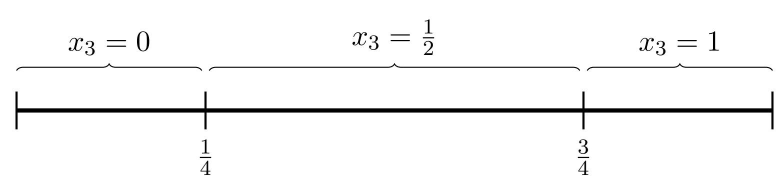


3.1.3 Third Period

Finally, in the third period, any politician in office will choose their most preferred policy. The voters take no actions. The behavior of politicians is illustrated in Figure 7.

$$\sigma_3^{*i} = \begin{cases} x_3 = 0 & \text{if } \hat{x}_i \in \left[0, \frac{1}{4}\right] \\ x_3 = \frac{1}{2} & \text{if } \hat{x}_i \in \left(\frac{1}{4}, \frac{3}{4}\right) \\ x_3 = 1 & \text{if } \hat{x}_i \in \left[\frac{3}{4}, 1\right] \end{cases}$$

Figure 7: *Any politician in the third period.*



Finally, Table 2 summarizes the voter's decisions given the politician in office and the policy chosen.

3.2 Propositions

I will now discuss the most notable implications of the model.

Proposition 1. *There exists a perfect Bayesian equilibrium in which:*

(i) a politician will be removed from office by the voter and subsequently reinstated, and

(ii) a politician may run for office and loose, then run a second time and win.

The equilibrium proof is provided in the appendix. This paper focuses primarily on the first half of this proposition; the second half follows. The changing state of the world allows for the realignment of the voter's preference. For a politician to serve non-consecutive terms, she must stick to her guns during her first term, choosing her ideal policy. This behavior is inline with the results of other models of career politicians that claim dedication to policy initiatives separates those who become career politicians from those with political careers.

Proposition 2. *A voter will remove a first period politician from office only if that politician chooses the opposite state of the world.*

While proposition 2 may seem intuitive, it is not immediately obvious that the voter would prefer to elect a politician whose policy choice matches *neither* state of the world over a random challenger. Though a challenger is untested, she has observed the state of the world in the first period and will update her strategy accordingly based on her ideal point and desire for re-election. The distribution of politicians in the first period who choose one half is symmetric about one half. By sheer number of politicians of who could choose the correct state of the world in the second period, electing a random challenger seems to be the obvious choice.

Just as the challenger adjusts her strategy after the state of the world is revealed, the incumbent does as well. With respect to the size of the group of ideal points, the likelihood of a first period incumbent choosing the correct state of the world in the second period is higher than that of a random challenger. Thus, the voter prefers to stick with the incumbent.

The implication regarding the behavior of politicians by proposition 2 is also noteworthy. Since choosing a moderate position will ensure re-election while choosing an extreme policy will may result in removal from office, a significant portion of politicians will choose one half. In fact, only politicians with ideal points close to the extremes will be unwilling to compromise. This pattern can be seen in Figure 4. The cut-points, α_1 and β_1, are highly dependent on the size of the office benefit and are to the left and right of one quarter and three quarters respectively. The ideal points one quarter and three quarters are significant because they delineate politicians whose first choice policy is zero and one, respectively, from those whose first choice

Table 2: *These tables shows the voter's choices in equilibrium based on the politician in office and the policy choice; policy choice* $\Rightarrow$ *voter choice. Table 2a shows the voter's choice at the end of the first period when the state of the world is revealed as* ω_1*. Table 2b contains two rows. The top shows the voter's choice when the state of the world in the second period is the same as the state of the world in the first period, while the bottom row shows the voter's choice when the state of the world in the second period is revealed to be different than the state of the world in the first period.*

(a) *First Period Voter Choices*

		Politician in Office
		M
State of the World	ω_1	$x_1 = \omega_1 \Rightarrow M$
		$x_1 = \frac{1}{2} \Rightarrow M$
		$x_1 = -\omega_1 \Rightarrow N$

(b) *Second Period Voter Choices*

		Politician in Office	
		M	***N***
State of the World	$\omega_2 = \omega_1$	$x_2 = \omega_1 \Rightarrow M$	$x_2 = \omega_1 \Rightarrow N$
		$x_2 = \frac{1}{2} \Rightarrow P$	$x_2 = \frac{1}{2} \Rightarrow P$
		$x_2 = -\omega_1 \Rightarrow P$	$x_2 = -\omega_1 \Rightarrow P$
	$\omega_2 = -\omega_1$	$x_2 = \omega_1 \Rightarrow P$	$x_2 = \omega_1 \Rightarrow M$
		$x_2 = \frac{1}{2} \Rightarrow P$	$x_2 = \frac{1}{2} \Rightarrow M$
		$x_2 = -\omega_1 \Rightarrow P$	$x_2 = -\omega_1 \Rightarrow M$

policy is one half. Politician from all groups choose one half, but only politicians whose first preference policy is an extreme will choose an extreme policy. The behavior of politicians with who choose an extreme policy is crucial to the following proposition.

Proposition 3. *In equilibrium, if a random challenger was elected to office for the second period and the state of the world changes, the voter will re-elect the first period incumbent regardless of the policy choice of the second period incumbent.*

The intuition for proposition 3 is as follows. The first period incumbent was removed from office for choosing the incorrect state of the world. The voter knows the policy the first period incumbent chose is the policy closest to her ideal point. If the state of the world changes from the first to the second period, it now matches the first period incumbent's policy choice. The state of the world is unlikely to change. The voter knows if the first period incumbent is elected for the third period, the politician will choose her first preference policy, which now matches the state of the world.

Notably, the voter elect the first period incumbent *even if the second period incumbent matches the state of the world after it changes.* In equilibrium, the voter is indifferent the first and second period incumbent. Both politicians have chosen the policy sincerely as they know they are more likely to be removed from office than re-elected. Traditionally in formal models of elections, when the voter is indifferent, the current incumbent is re-elected; however, in this set up, the voter must remove the current incumbent in favor of the first period incumbent. Otherwise, second period incumbents with ideal points close to one quarter or three quarters whose first preference policy is actually one half will compromise and choose either zero or one, respectively, in an effort to be re-elected.

This behavior can be seen in Figure 5 when the first period incumbent is re-elected. If the voter went on to re-elect the second period incumbent after the state of the world changed, there would be some positive probability the politician would actually choose the moderate policy. The first period incumbent, on the other hand, will choose a possible state of the world with probability one. As seen in Figure 6, then, the voter's decision to only re-elect the first period incumbent causes all politicians with ideal points to the right[5] of one half to act sincerely. Politicians never choose their least preferred policy. Second period incumbents whose least preferred policy was the state of the world in the first period know they have no chance at re-election, so they choose their most preferred policy.

Furthermore, Proposition 3 details the *only* circumstances under which a politician who was removed from office may be brought back. If the state of the world does not change, the voter is justified in removing the first period politician and still does believes the state of the world is unlikely to change. If the politician in the second period chooses the incorrect state of the world and the sate of the world does not change, the voter takes her chances with a random challenger.

[5] When the state of the world in the first period is zero, as in the example.

3.3 Comparative Statics

Proposition 3 makes for an interesting comparative static. The more likely the state of the world is to change, the more likely an incumbent who was previously removed from office is to return. Highly volatile countries should be more likely to see politicians who previously held office returning. Developing countries where the government has only weak control over territory provide excellent circumstances for politician to make a comeback after being removed from office.

Developing countries are not the only places leaders are removed and return. In fact, there are far more examples of leaders in developing countries not returning after being removed from office. Also, the introduction referenced the number of non-consecutive executives in OECD countries, include a large number of Westernized, fully developed countries. The likelihood of returning politicians is not only a result of a country's volatility, but also the size of the office benefit. The likelihood of compromising to a moderate policy choice in the first increases as the size of the office benefit increases, resulting in fewer politicians taking extreme stances. Politicians who have more outside options or whose power and influence are not strictly a consequence of their office can afford to take more extreme positions.

4 Conclusion

This paper has attempted to explain the decision making process of voters who remove a politician from office only to reinstate that politician at a later time. The changing state of the world causes the voter to occasionally update her preferences. The dichotomous states of the world coupled with the continuum of politician ideal points allows for the shifting of politician strategies, resulting in the voter updating her beliefs about politicians' ideal points and policy choices. Reinstatement of a politician only occurs if the voters change their preferences, which according to this model, happens when the state of the world changes. Furthermore, if the state of the world does change, voters will remove the current politician regardless of her policy choice. This behavior is reminiscent of the saying " better the devil you know than the devil you don't."

The model not only addresses an understudied phenomenon, but also accounts for the phenomenon's relative rarity. It is crucial that the voter place more weight on the state of the world remaining the same even after witnessing a change. Just as crucial to a politician's reinstatement is that the state of the world does in fact change. With only a sufficiently small probability of the state of the world changing, there is only sufficiently small probability of a politician returning to office.

Objections may be made to the candidate-centric construction of the model. Strategic choices within party systems offer an alternative explanation. Parties may be strategically choosing which candidates to pit against each other, significantly decreasing both the role a politician's policy choice and the voter's expectations by limiting available options.

I allow that parties are certainly a force to consider and this is a viable alternative explanation; however, the primary goal of this model is to gain basic insights into a previously undiscussed pattern of behavior. When attempting to model a new pattern, it is prudent to begin by addressing the problem in the simplest way possible. Constructing a model with fewer actors possessing uncomplicated utilities in the least amount of periods allows for scholars to first see what underlying mechanism are at work before the process becomes too complicated. Addressing the reinstatement of politicians when parties are at play is certainly viable work for the future, but not a starting point.

As stressed throughout the paper, the dynamics modeled here are quite novel, implying there remains plenty of room to experiment with different constructions and specifications. In particular, I would like to address the out-of-office utility of politicians. The way the utilities are structured here imply politicians are only sensitive to policy choices when they are in office. Their out-of-office payoffs are stationary. For the purposes of this paper, assuming out-of-office payoffs to be zero simplified calculations. In the future, I would like to be able to lessen the differences between politicians and the voter, beginning with utility constructions.

Additionally, I hope to examine the relationship between the available states of the world and policy choices. Keeping dichotomous states of the world but allowing for a continuum of policy choices results in an extremely fragile equilibrium which does not survive standard refinements. Reducing the policy choices to two which match the states of the world demands an increased office benefit to induce politicians to run if an option is too far from their ideal point. The larger office benefit results in politicians potentially compromising to their least preferred ideal point, reducing the likelihood of seeing a politician being thrown out of office in the first place. While three policy options do allow for the desired behavior, the question remain whether there exists some relationship between the number of state of the world and the number of policy choices.

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Appendices

Before I begin constructing the proof of the equilibrium, I remind the reader of the utility functions of the actors, the timing of the game, and notation.

Politician i's Utility: $u_i(x_t|\hat{x}) = -|x_t - \hat{x}_i| + B$, $B \in \left(\frac{1}{4}, \frac{1}{2}\right)$

Voter v's Utility: $u_v(x_t|\omega_t) = -|\omega - x_t|$

Table 3: *Preference orderings by group*

Group	x'	x''	x'''
A	0	$\frac{1}{2}$	1
B	$\frac{1}{2}$	0	1
C	$\frac{1}{2}$	1	0
D	1	$\frac{1}{2}$	0

Time-line:

1. First period:
   (a) Nature chooses a state of the world.
   (b) The politician in office chooses a policy.
   (c) The state of the world is revealed.
   (d) The voter decides which politician holds office in the next period.
2. Second period:
   (a) Nature chooses a state of the world, possibly different than the previous state.
   (b) The politician in office chooses a policy.
   (c) The state of the world is revealed.
   (d) The voter decides which politician holds office in the next period.
3. Third period:
   (a) Nature chooses a state of the world, possibly different from the previous state.
   (b) The politician in office chooses a policy.
   (c) The state of the world is revealed.

Notation: If a politician holds office, that politician is then referred to as the incumbent from the period they first held office. The first period politician is M. If a new politician is elected to hold office in the second period, that politician is N.

Random challengers are indexed by the period in which they would assume office if elected. The random challenger the first period incumbent M faces is N. The random challenger the politician in office during the second period faces is P.

A Statement of Full Equilibrium

First Period:

$$\sigma_1^{*M} = \begin{cases} x_1 = 0 & \text{if } \hat{x}_M \in [0, \alpha_1] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_M \in (\alpha_1, \beta_1) \\ x_1 = 1 & \text{if } \hat{x}_M \in [\beta_1, 1] \end{cases}$$

$$\mu^*_{\omega_1=0} = \frac{1}{2} \qquad \mu^*_{\omega_1=1} = \frac{1}{2}$$

$$\sigma_1^{*v} = \begin{cases} M & \text{if } x_1 = \omega_1 \\ M & \text{if } x_1 = \frac{1}{2} \\ N & \text{if } x_1 = -\omega_1 \end{cases}$$

$$\mu^*_{1,\hat{x}_M}(x_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{1,\hat{x}_M}(x_1 = \frac{1}{2}) \sim U(\alpha_1, \beta_1) \quad \mu^*_{1,\hat{x}_M}(x_1 = 1) \sim U[\beta_1, 1]$$

Second period:

When M is in office:

$$\sigma_2^{*M}(\omega_1) = \begin{cases} x_1 = 0 & \text{if } \hat{x}_M \in [0, \alpha_{2,\omega_1}] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_M \in (\alpha_{2,\omega_1}, \beta_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_M \in [\beta_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1 - \rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(M, \omega_2 = \omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(M, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases}$$

Beliefs following equilibrium actions:

$$\mu^*_{2,\hat{x}_M}(x_2 = 0 | M, x_1 = 0, \omega_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_M}\left(x_2 = \frac{1}{2} | M, x_1 = 0, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 1 | M, x_1 = 0, \omega_1 = 0) = 1$$

$$\mu^*_{2,\hat{x}_M}(x_2 = 0 | M, x_1 = 1, \omega_1 = 1) = 0 \quad \mu^*_{2,\hat{x}_M}\left(x_2 = \frac{1}{2} | M, x_1 = 1, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 1 | M, x_1 = 1, \omega_1 = 1) \sim U\left[\beta_1, 1\right]$$

$$\mu^*_{2,\hat{x}_M}(x_2 = 0 | M, x_1 = \frac{1}{2}) \sim U[\alpha_1, \alpha_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_M}\left(x_2 = \frac{1}{2} | M, x_1 = \frac{1}{2}\right) \sim U\left(\alpha_{2,\omega_1}, \beta_{2,\omega_1}\right)$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 1 | M, x_1 = \frac{1}{2}) \sim U\left[\beta_{2,\omega_1}, \beta_1\right]$$

Beliefs following off-the-path actions:

$$\mu^*_{2,\hat{x}_M}(x_2 = 0 | M, x_1 = 0, \omega_1 = 1) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_M}\left(x_2 = \frac{1}{2} | M, x_1 = 0, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 1 | M, x_1 = 0, \omega_1 = 1) = 1$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 0 | M, x_1 = 1, \omega_1 = 0) = 0 \quad \mu^*_{2,\hat{x}_M}\left(x_2 = \frac{1}{2} | M, x_1 = 1, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_M}(x_2 = 1 | M, x_1 = 1, \omega_1 = 0) \sim U\left[\beta_1, 1\right]$$

When N is in office:

$$\sigma_2^{*N}(\omega_1)\begin{cases} x_2 = 0 & \text{if } \hat{x}_N \in [0, \alpha'_{2,\omega_1}] \\ x_2 = \frac{1}{2} & \text{if } \hat{x}_N \in (\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_N \in [\beta'_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1-\rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(N, \omega_2 = \omega_1) = \begin{cases} N & \text{i f } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(N, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ M & \text{if } x_2 = \frac{1}{2} \\ M & \text{if } x_2 = -\omega_2 \end{cases}$$

$$\mu^*_{2,\hat{x}_N}(x_2|N) \sim U[0, \alpha'_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_N}(x_2|N) \sim U(\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \quad \mu^*_{2,\hat{x}_N}(x_2 = 1|N) \sim U[\beta'_{2,\omega_1}, 1]$$

$$\alpha'_{2,0} = \frac{2(1-\rho)B + \rho}{2+2\rho} \qquad \beta'_{2,0} = \frac{3}{4}$$

$$\alpha'_{2,1} = 1 - \beta'_{2,0} \qquad \beta'_{2,1} = 1 - \alpha'_{2,0}$$

Third period:

$$\sigma_3^{*i} = \begin{cases} x_3 = 0 & \text{if } \hat{x}_i \in \left[0, \frac{1}{4}\right] \\ x_3 = \frac{1}{2} & \text{if } \hat{x}_i \in \left(\frac{1}{4}, \frac{3}{4}\right) \\ x_3 = 1 & \text{if } \hat{x}_i \in \left[\frac{3}{4}, 1\right] \end{cases}$$

$$\mu^*_{\omega_3=\omega_2} = 1-\rho \qquad \mu^*_{\omega_3=-\omega_2} = \rho$$

In order for the proposed equilibrium to satisfy the conditions of a perfect Bayesian equilibrium the strategies must be sequentially rational given the beliefs and the beliefs must be consistent given the strategies. To construct the proof for the equilibrium, I begin in the third period.

B Equilibrium Proof

The model is solved via backwards induction. The proof follows.

B.1 Third Period

The game ends at the conclusion of the third period. There is no election in the third period so there is no action for the voter to take. The politician in office then chooses the policy closest to her ideal point as there exists no incentives to act to the contrary. Politicians with ideal points between $\left[0, \frac{1}{4}\right]$ choose $x_3 = 0$. Politicians with ideal points between $\left(\frac{1}{4}, \frac{3}{4}\right)$ choose $x_3 = \frac{1}{2}$. Politicians with ideal points between $\left[\frac{3}{4}, 1\right]$ choose $x_3 = 1$.

The third period strategies can be written as follows:

$$\sigma_3^{*i} = \begin{cases} x_3 = 0 & \text{if } \hat{x}_i \in \left[0, \frac{1}{4}\right] \\ x_3 = \frac{1}{2} & \text{if } \hat{x}_i \in \left(\frac{1}{4}, \frac{3}{4}\right) \\ x_3 = 1 & \text{if } \hat{x}_i \in \left[\frac{3}{4}, 1\right] \end{cases}.$$

The voter knows the politician holding office in the third period chooses her ideal policy regardless of the state of the world. Additionally, the voter knows the politicians are uniformly distributed on the interval $[0, 1]$ according to their ideal points. Should the voter elect a random challenger to the third period, P, the politician will not have taken any actions to inform the voter of what her ideal point might be. The voter's beliefs regarding which action a random challenger will take in the third period can then be written as follows:

$$\mu^v_{\hat{x}_i}\left(x_3 = 0\right) \sim U\left[0, \frac{1}{4}\right]$$
$$\mu^v_{\hat{x}_i}\left(x_3 = \frac{1}{2}\right) \sim U\left[\frac{1}{4}, \frac{3}{4}\right]$$
$$\mu^v_{\hat{x}_i}\left(x_3 = 1\right) \sim U\left[\frac{3}{4}, 1\right]$$

Let $\alpha_3 \equiv \frac{1}{4}$ such that every politician with an ideal point to the left of α_3 choose $x_3 = 0$. Similarly, let $\beta_3 \equiv \frac{3}{4}$ such that every politician with an ideal point to the right of β_3 choose $x_3 = 1$. All politicians with ideal points between α_3 and β_3 choose $x_3 = \frac{1}{2}$.

With the voter's beliefs and the third period politician's strategy specified, I can now calculate the voter's expected utility from electing a random challenger.

$$Eu_v(P|\omega_2) = \frac{1}{4}u_v(0|\omega_2) + \frac{1}{2}u_v\left(\frac{1}{2}|\omega_2\right) + \frac{1}{4}u_v(1|\omega_2)$$

The voter does not know for certain what the state of the world will be in the final period. I am able to calculate the the expected per period utility of the voter given the previous state of the world.

Given the state of the world is zero:

$$\begin{aligned} u_v(0|0) =&(1-\rho)[-(0-0)] + \rho[-(1-0)] \\ =& -\rho \end{aligned}$$

$$\begin{aligned} u_v(1|0) =&(1-\rho)[-(0-1)] + \rho[-(1-1)] \\ =& -1+\rho \end{aligned}$$

$$\begin{aligned} u_v\left(\frac{1}{2}|0\right) =&(1-\rho)[-(0-\frac{1}{2})] + \rho[-(1-\frac{1}{2})] \\ =& -\frac{1}{2} \end{aligned}$$

Given the state of the world is one:

$$\begin{aligned} u_v(0|1) =&(1-\rho)[-(1-0)] + \rho[-(1-1)] \\ =& -1+\rho \end{aligned}$$

$$\begin{aligned} u_v(1|1) =&(1-\rho)[-(1-1)] + \rho[-(0-1)] \\ =& -\rho \end{aligned}$$

$$\begin{aligned} u_v\left(\frac{1}{2}|1\right) =&(1-\rho)[-(1-\frac{1}{2})] + \rho[-(0-\frac{1}{2})] \\ =& -\frac{1}{2} \end{aligned}$$

The results of these calculations are summarizes in Table 4.

Table 4: *The expected utility for a voter given what the state of the world was revealed to be in the previous period and the current period policy choice.*

x_t	$\omega_{-t}=0$	$\omega_{-t}=1$
0	$-\rho$	$-1+\rho$
$\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
1	$-1+\rho$	$-\rho$

Using the results from Table 4, I am able to calculate the voter's expected utility from electing a random challenger to office in the third period. Without loss of generality, assume the state of the world in the previous period was zero, $\omega_2 = 0$.

$$
\begin{aligned}
Eu_v(RC|\omega_2 = 0) =& \frac{1}{4}u_v(0) + \frac{1}{2}u_v\left(\frac{1}{2}\right) + \frac{1}{4}u_v(1) \\
=& \frac{1}{4}\{-\rho\} - \frac{1}{2}\left\{\frac{1}{2}\right\} + \frac{1}{4}\{-(1-\rho)\} \\
=& -\frac{1}{2}
\end{aligned}
$$

The expected utility does not depend on the state of the world; therefore, regardless of the state of the world, the expected utility of the voter when electing a random challenger for the third period is $-\frac{1}{2}$.

Equilibrium play for the third period has been identified as well as the voter's expected utility from electing a random challenger to office in the third period. The strategies and beliefs of the actors in the second period rely on this information.

B.2 Second Period

B.2.1 Random Challenger

Conditional on a random challenger being elected to office for the second period, the proposed equilibrium calls for the voter to re-elect the second period incumbent, N, if the politician chooses the correct state of the world *and* the state of the world remains the same as it was in the first period.

In the case of a random challenger being elected to office for the second period, the new incumbent, N, has observed the state of the world in the first period and knows it only changes with probability ρ, sufficiently close to zero. Furthermore, the politician also knows the voter only re-elects her if $x_2 = \omega_1$ and the state of the world does not change. The politicians then play a cut-point strategy based on their ideal point. The ideal points at which the politician is indifferent between re-election and choosing her most preferred policy provides the cut-points. All politicians to the left of the politician indifferent between $x_2 = 0$ and $x_2 = \frac{1}{2}$ choose $x_2 = 0$. All politicians to the right of the politician indifferent between choosing $x_2 = \frac{1}{2}$ and $x_2 = 1$ choose $x_2 = 1$. Politicians with ideal points between the indifferent politicians choose $x_2 = \frac{1}{2}$.

As politicians from different groups have different preference orders (see Table ??), I check indifference conditions by group. Without loss of generality assume the sate of the world in the first period was zero, $\omega_1 = 0$.

Group A

$$\begin{aligned} Eu_A(x'|\omega_1 = 0) &= -|x' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B] \\ &= -(\hat{x}_i - 0) + B + (1-\rho)[-(\hat{x}_i - 0) + B] \\ &= (-2+\rho)\hat{x}_i + (2-\rho)B \end{aligned}$$

$$\begin{aligned} Eu_A(x''|\omega_1 = 0) &= -|x'' - \hat{x}_i| + B \\ &= -(\frac{1}{2} - \hat{x}_i) + B \\ &= \hat{x}_i + B - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu_A(x'''|\omega_1 = 0) &= -|x''' - \hat{x}_i| + B \\ &= -(1 - \hat{x}_i) + B \\ &= \hat{x}_i + B - 1 \end{aligned}$$

Now to see who compromises.

$$\begin{aligned} Eu_A(x'|\omega_1 = 0) &\geq Eu_A(x''|\omega_1 = 0) \\ (-2+\rho)\hat{x}_i + (2-\rho)B &\geq \hat{x}_i + B - \frac{1}{2} \\ \hat{x}_i &\leq \frac{2(1-\rho)B + 1}{2(3-\rho)} \end{aligned} \tag{3}$$

As ρ approaches zero, the right hand side of the inequality approaches $\frac{2B+1}{6}$. As B varies within $(\frac{1}{4}, \frac{1}{2})$, the limit varies withing $(\frac{1}{4}, \frac{1}{3})$. All politicians in group A have ideal points less than or equal to $\frac{1}{4}$, so no politician in group A compromises to $x = x''$. Since no politicians in this group are choosing $x = x''$ and politicians have strict preference orderings, no politician chooses x'''. No politician chooses $x = x'''$ over $x = x'$. Therefore, all politicians in group A always choose $x = x'$ in the second period.

Group B

$$\begin{aligned} Eu_B(x'|\omega = 0) &= -|x' - \hat{x}_i| + B \\ &= -(\frac{1}{2} - \hat{x}_i) + B \\ &= \hat{x}_i + B - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu_B(x''|\omega_1 = 0) &= -|x'' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B] \\ &= -(\hat{x}_i - 0) + B + (1-\rho)[-(\frac{1}{2} - \hat{x}_i) + B] \\ &= -\rho\hat{x}_i + (2-\rho)B - \frac{1}{2} + \frac{1}{2}\rho \end{aligned}$$

$$\begin{aligned} Eu_B(x'''|\omega_1 = 0) &= -|x''' - \hat{x}_i| + B \\ &= -(1 - \hat{x}_i) + B \\ &= \hat{x}_i + B - 1 \end{aligned}$$

Now I check for compromises.

$$\begin{aligned} Eu_B(x'|\omega = 0) &\geq Eu_B(x''|\omega_1 = 0) \\ \hat{x}_i + B - \frac{1}{2} &\geq -\rho\hat{x}_i + (2-\rho)B - \frac{1}{2} + \frac{1}{2}\rho \\ \hat{x}_i &\geq \frac{2(1-\rho)B + \rho}{2 + 2\rho} \end{aligned} \tag{4}$$

This is the divide along which politicians of group B splits. Those who have ideal points which satisfy the strict inequality choose $x = x'$, while those whose ideal points are less than or equal to the right hand side of the inequality chooses $x = x''$.

As ρ approaches zero, the right hand side of the inequality approaches B from the left. As the state of the world becomes less likely to change, more politicians are willing to compromise in order to be re-elected. Additionally, the larger the office benefit, the more politicians compromise.

The next step is to check if any of the compromising politicians compromise further to $x = x'''$.

$$\begin{aligned} Eu_B(x''|\omega_1 = 0) &\geq Eu_B(x'''|\omega_1 = 0) \\ -\rho\hat{x}_i + (2-\rho)B - \frac{1}{2} + \frac{1}{2}\rho &\geq \hat{x}_i + B - 1 \\ \hat{x}_i &\leq \frac{2(1-\rho)B + 1 + \rho}{2 + 2\rho} \end{aligned}$$

As ρ approaches zero, the right hand side of the inequality approaches $B + \frac{1}{2}$ from the right. With B varying within $(\frac{1}{4}, \frac{1}{2})$, the limit of the inequality varies within $(\frac{1}{2}, 1)$. The largest ideal point of politicians in group B is $\hat{x} = \frac{1}{2}$; therefore, no politicians in group B chooses $x = x'''$ over $x = x''$.

Now set $\alpha'_{2,0} = \frac{2(1-\rho)B+\rho}{2+2\rho}$, from equation 2. Politicians with ideal points less than or equal to $\alpha'_{2,0}$ choose $x = 0$.

Group C

$$\begin{aligned} Eu_C(x'|\omega_1 = 0) &= -|x' - \hat{x}_i| + B \\ &= -(\hat{x}_i - \frac{1}{2}) + B \\ &= -\hat{x}_i + B + \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu_C(x''|\omega_1 = 0) &= -|x'' - \hat{x}_i| + B \\ &= -(1 - \hat{x}_i) + B \\ &= \hat{x}_i + B - 1 \end{aligned}$$

$$\begin{aligned} Eu_C(x'''|\omega_1 = 0) &= -|x''' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B] \\ &= -(\hat{x}_i - 0) + B(1-\rho)[-(\hat{x}_i - \frac{1}{2}) + B] \\ &= (-2+\rho)\hat{x}_i + (2-\rho)B + \frac{1}{2} - \frac{1}{2}\rho \end{aligned}$$

Now to see who compromises.

$$
\begin{aligned}
Eu_C(x'|\omega_1 = 0) &\geq Eu_C(x''|\omega_1 = 0) \\
-\hat{x}_i + B + \frac{1}{2} &\geq \hat{x}_i + B - 1 \\
\hat{x}_i &\leq \frac{3}{4}
\end{aligned} \tag{5}
$$

All politicians in group C have ideal points less than $\frac{3}{4}$; therefore no politicians in group C choose $x = x''$ over $x = x'$. Since no politicians are choosing x'', no politicians choose x'''. If a second period politician is in group C, she always chooses $x = x'$.

Group D

$$
\begin{aligned}
Eu_D(x'|\omega_1 = 0) &= -|x' - \hat{x}_i| + B \\
&= -(1 - \hat{x}_i) + B \\
&= \hat{x}_i + B - 1
\end{aligned}
$$

$$
\begin{aligned}
Eu_D(x''|\omega_1 = 0) &= -|x'' - \hat{x}_i| + B \\
&= -(\hat{x}_i - \frac{1}{2}) + B \\
&= -\hat{x}_i + B + \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
Eu_D(x'''|\omega_1 = 0) &= -|x''' - \hat{x}_i| + B + (1 - \rho)[-|x''' - \hat{x}_i| + B] \\
&= -(\hat{x}_i - 0) + B + (1 - \rho)[-(1 - \hat{x}_i) + B] \\
&= -\rho\hat{x}_i + 2B - 1 + \rho
\end{aligned}
$$

Now I check for compromises.

$$
\begin{aligned}
Eu_D(x'|\omega_1 = 0) &\geq Eu_D(x''|\omega_1 = 0) \\
\hat{x}_i + B - 1 &\geq -\hat{x}_i + B + \frac{1}{2} \\
\hat{x}_i &\geq \frac{3}{4}
\end{aligned} \tag{6}
$$

All politicians in group D have ideal points which are greater than or equal to $\frac{3}{4}$; therefore, no politicians in group D compromise. All politicians in group D always choose $x = x'$.

I then set $\beta'_{2,0} = \frac{3}{4}$. The voter believes all politicians with ideal points to the right of $\beta'_{2,0}$ choose $x = 1$, which is consistent with the politician's strategy.

To summarize the work in the previous subsections, all politicians with ideal points to the left of $\alpha'_{2,0} = \frac{2(1-\rho)B+\rho}{2+2\rho}$ choose $x_2 = 0$, all politicians with ideal points to the right of $\beta'_{2,0} = \frac{3}{4}$ choose $x_2 = 1$, and all politicians in between $\alpha'_{2,0}$ and $\beta'_{2,0}$ choose $x_2 = \frac{1}{2}$.

The cut-points in this section were determined with respect to the state of the world in the first period defined as zero, $\omega_1 = 0$. If the state of the world had been revealed to be one, ω_1, the cut-points would be "reversed" with respect to one. That is, $\alpha'_{2,1} = 1 - \beta'_{2,1}$ and $\beta'_{2,1} = 1 - \alpha'_{2,0}$.

Voter Expectation The voter's strategy is also optimal given the actions of the politician. The voter knows the politician plays a cut-point strategy where politicians to the left of α'_{2,ω_1} choose $x_2 = 0$, politicians to the right of β'_{2,ω_1} choose $x_2 = 1$, and politicians with ideal points in between α'_{2,ω_1} and β'_{2,ω_1} choose $x_2 = \frac{1}{2}$.

$$\begin{aligned}
\mu^v_{\hat{x}_i}(x_2 = 0, \omega_1 = 0) &\sim U\left[0, \alpha_{2,\omega_1}\right] \\
\mu^v_{\hat{x}_i}(x_2 = \frac{1}{2}, \omega_1 = 0) &\sim U\left(\alpha_{2,\omega_1}, \beta_{2,\omega_1}\right) \\
\mu^v_{\hat{x}_i}(x_2 = 1, \omega_1 = 0) &\sim U\left[\beta_{2,\omega_1}, 1\right]
\end{aligned}$$

The voter knows politician elected to the third period chooses the policy closest to her own ideal point.

These cut-points do not vary with the state of the world in the second period. The politician in office does not know for certain the state of the world in the current period, only the state of the world in the last period. Though the voter knows the state of the world in the current period once a policy has been chosen, it should not affect her beliefs regarding the ideal point of the politician.

In equilibrium then, it must the voter is better off re-electing a politician who chooses the correct state of the world when the state of the world does not change rather than electing a random challenger, but worse off if re-electing a random challenger who chooses $x_2 = \frac{1}{2}$ or the wrong state of the world.

First, I calculate the voter's expected utility from electing a politician who chose each action. Without loss of generality, assume the state of the world is zero, $\omega_2 = 0$, for the rest of the analysis in this section.

$$Eu_v(N|x_2 = 0, \omega_2 = 0) = \frac{1}{\alpha'_{2,0}} \left[\frac{1}{4} u_v(0|0) + \left(\alpha'_{2,0} - \frac{1}{4} \right) u_v \left(\frac{1}{2}|0 \right) \right]$$
$$= \frac{2+2\rho}{2(1-\rho)B+\rho} \left(\frac{1}{8} - \frac{1}{4}\rho \right) - \frac{1}{2}$$

$$Eu_v(N|x_2 = 1, \omega_2 = 0) = u_v(1|0)$$
$$= -1 + \rho$$

$$Eu_v \left(N|x_2 = \frac{1}{2}, \omega_2 = 0 \right) = u_v \left(\frac{1}{2}|0 \right)$$
$$= -\frac{1}{2}$$

Now I compare the expected utilities from re-electing a second period incumbent to the voter's expected utility from electing a random challenger.

$$Eu_v(P) \leq Eu_v(N|x_2 = 0, \omega_2 = 0)$$
$$-\frac{1}{2} \leq \frac{2+2\rho}{2(1-\rho)B+\rho} \left(\frac{1}{8} - \frac{1}{4}\rho \right) - \frac{1}{2}$$
$$0 \leq \frac{2+2\rho}{2(1-\rho)B+\rho} \left(\frac{1}{8} - \frac{1}{4}\rho \right)$$

As ρ approaches zero, the right hand side of the inequality approaches $\frac{1}{8B}$ from the the right, thus the inequality holds strictly. It is always better for the voter to re-elect the second period incumbent instead of the random challenger when $x_2 = \omega_2$.

It also must be true that the voter elects the second period incumbent instead of the first period incumbent, M. In equilibrium, the voter only rejects the first period incumbent when the $x_1 = -\omega_1$ and the voter believes any politician who chooses $x_1 = -\omega_1$ in the first period plays the same policy in the last period, $x_3 = -\omega_1$. Under these assumptions (to be proved later in the paper) and the current assumption that the state of the world remained the same, the voter's expected utility from re-electing the first period incumbent for the third period would be

$$Eu_v(M|x_1 = 1, \omega_1 = 0, \omega_2 = 0) = u(1|0) = -1 + \rho$$

which is strictly less than $-\frac{1}{2}$ therefore strictly less than the voter's expected utility from re-electing the second period incumbent, N.

Next, in equilibrium the voter elects a random challenger, P to office the third period if the second period incumbent, N, chooses $x_2 = 1$ or $x_2 = \frac{1}{2}$. Then it must be the voter's expected utility from electing the random challenger P is at least as big as her expected utility from re-electing the second period incumbent, N.

$$\begin{aligned} Eu_v(P) &\geq Eu_v(N|x_2 = 1, \omega_2 = 0) \\ -\frac{1}{2} &\geq (-1 + \rho) \end{aligned}$$

As ρ is assumed to be close to zero, the inequality holds strictly. The voter elects a random challenger P over the second period incumbent, N when $x_2 = -\omega_2$.

$$\begin{aligned} Eu_v(P) &\geq Eu_v(N|x_2 = \frac{1}{2}, \omega_2 = 0) \\ -\frac{1}{2} &\geq -\frac{1}{2} \end{aligned}$$

Clearly, the inequality is true. The voter is indifferent between re-electing the second period incumbent and a random challenger. In equilibrium, the voter chooses the random challenger.

Next I check that the voter's actions are optimal when the state of the world in the second period is different from the state of the world in the first period. As previously noted, the cut-points remain the same. Only the voter's expected utilities change.

In equilibrium, if a second period incumbent, N, is in office and the state of the world in the first period is revealed to be different than the state of the world in the second period, the voter always re-elects the first period incumbent,M, regardless of the choice of policy by the second period incumbent, N. Without loss of generality, Continue to assume the state of the first period state of the world is zero.

As per the (brief) previous discussion, the voter only removes the first period incumbent is $x_1 = -\omega_1$ and believes the politician who chooses $x_1 = -\omega_1$ chooses $x_3 = -\omega_1$. Under the assumption the state of the world in the first period was zero, $\omega_1 = 0$, that means the first period incumbent chose $x_1 = 1$. Now that the state of the world has changed, the voter's expected utility from re-electing the first period incumbent is

$$Eu_v(M|x_1 = 1, \omega_1 = 0, \omega_2 = 1) = u_v(1|1) = -\rho.$$

Now I compare the voter's utility from re-electing the first period incumbent, M, for the third period to re-electing the second period incumbent, N, or a random challenger, P.

First I calculate the voter's expected utility from re-electing a second period incumbent when the state of the world has changed.

$$
\begin{aligned}
Eu_v(N|x_2 = 0, \omega_2 = 1) &= \frac{1}{\alpha'_{2,0}} \left[\frac{1}{4} u_v(0|1) + \left(\alpha'_{2,0} - \frac{1}{4} \right) u_v \left(\frac{1}{2}|1 \right) \right] \\
&= \frac{2 + 2\rho}{2(1-\rho)B + \rho} \left(\frac{1}{4}\rho - \frac{1}{8} \right) - \frac{1}{2} \\
Eu_v(N|x_2 = 1, \omega_2 = 1) &= u_v(1|1) \\
&= -\rho \\
Eu_v \left(N|x_2 = \frac{1}{2}, \omega_2 = 1 \right) &= u_v \left(\frac{1}{2}|1 \right) \\
&= -\frac{1}{2}
\end{aligned}
$$

The voter's expected utility from electing a random challenger remains the same. Now I compare the voter's expected utilities.

$$
\begin{aligned}
Eu_v(M|x_1 = 1, \omega_1 = 0, \omega_2 = 1) &\geq Eu_v(N|x_2 = 0, \omega_2 = 1) \\
-\rho &\geq \frac{2 + 2\rho}{2(1-\rho)B + \rho} \left(\frac{1}{4}\rho - \frac{1}{8} \right) - \frac{1}{2}
\end{aligned}
$$

Note that the term in parenthesis on the right hand side of the inequality is negative. Then, right side of the inequality is a negative minus a negative. Again, ρ is sufficiently close to zero and thus $-\rho > -\frac{1}{2}$. The right hand side is strictly less than $-\frac{1}{2}$ so the inequality holds strictly. The voter would rather re-elect the first period incumbent, M, instead of the second period incumbent when $x_2 = \omega_1$ and the state of the world has changed.

$$
\begin{aligned}
Eu_v(M|x_1 = 1, \omega_1 = 0, \omega_2 = 1) &\geq Eu_v(N|x_2 = 1, \omega_2 = 1) \\
-\rho &\geq -\rho
\end{aligned}
$$

Clearly, the equation holds. The voter is indifferent between re-electing the first or second periond incumbent. In equilibrium then, the voter elects the first period incumbent, M, when $x_2 = \omega_2$ and the state of the world has changed.

$$Eu_v(M|x_1 = 1, \omega_1 = 0, \omega_2 = 1) \geq Eu_v\left(N|x_2 = \frac{1}{2}, \omega_2 = 1\right)$$
$$-\rho \geq -\frac{1}{2}$$

As already noted, ρ is very close to zero and thus the inequality holds strictly. The voter would prefer to re-elect the first period incumbent, M, when $x_2 = \frac{1}{2}$ and the state of the world has changed.

Finally, I compare the expected utility of the voter from electing a first period incumbent, M, to the expected utility from electing a random challenger, P.

$$Eu_v\left(M|x_1 = 1, \omega_1 = 0, \omega_2 = 1\right) \geq Eu_v(P)$$
$$-\rho \geq -\frac{1}{2}$$

It has already been shown this inequality holds strictly.

If the state of the world changes from the first to the second period and the voter removed the first period incumbent from office, the voter always re-elects the first period incumbent to office for the third period.

I summarize formally the part of the equilibrium proved here, which is the second period actions when a random challenger is elected to hold office in the second period.

$$\sigma_2^{*N}(\omega_1)\begin{cases} x_2 = 0 & \text{if } \hat{x}_i \in [0, \alpha'_{2,\omega_1}] \\ x_2 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta'_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1-\rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(N, \omega_2 = \omega_1) = \begin{cases} N & \text{i f } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(N, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ M & \text{if } x_2 = \frac{1}{2} \\ M & \text{if } x_2 = -\omega_2 \end{cases}$$

$$\mu^*_{2,\hat{x}_i}(x_2|N) \sim U[0, \alpha'_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}(x_2|N) \sim U(\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \quad \mu^*_{2,\hat{x}_i}(x_2 = 1|N) \sim U[\beta'_{2,\omega_1}, 1]$$

$$\alpha'_{2,0} = \frac{2(1-\rho)B+\rho}{2+2\rho} \qquad \beta'_{2,0} = \frac{3}{4}$$

$$\alpha'_{2,1} = 1-\beta'_{2,0} \qquad \beta'_{2,1} = 1-\alpha'_{2,0}$$

B.2.2 Incumbent

The behavior of the first period incumbent re-elected to the second period depends in part on the actions she took in the first period. Similarly, the voter's beliefs over which actions the incumbent will take in the third period, should the voter choose to re-elect her, depend on the actions taken by the incumbent in the first period as well as the second period. This section is divided into three subsections; one for every possible first period choice.

$x_1 = \omega_1$ I begin by addressing the strategies of a re-elected first period incumbent who chose $x_1 = \omega_1$. In equilibrium, only politicians whose first choice policy is an extreme ($x_1 = 0$ or $x_1 = 1$) choose an extreme policy in the first period. Intuitively, if choosing $x_1 = \frac{1}{2}$ ensures re-election, and choosing a state of the world results in re-election with only some probability, only politicians who strongly prefer their first choice policy are willing to risk removal from office. Therefore, I safely assume $\alpha_1 < \frac{1}{4}$ and $\beta_1 > \frac{3}{4}$.

Politician Strategies:

To see how a politician who chose $x_1 = \omega_1$ behaves in the second period, assume $\omega_1 = 0$. In equilibrium, the voter only removes the incumbent from office if x_2 does not match the state of the world. If the politician chose $x_1 = 0$, then the politician belongs in group A as only politicians in group A have $x' = 0$, (see table ??).

$$\begin{aligned}
Eu(x = x'|\hat{x}_t \in A, \omega_1 = 0) &= -|x' - \hat{x}_t| + B + (1-\rho)[-|x' - \hat{x}| + B] \\
&= -(\hat{x}_t - 0) + B + (1-\rho)[-(\hat{x}_i - 0) + B] \\
&= -(2-\rho)\hat{x}_i + (2-\rho)B
\end{aligned}$$

$$\begin{aligned}
Eu(x = x''|\hat{x}_i \in A, \omega_1 = 0) &= -|x'' - \hat{x}_i| + B + 0 \\
&= -(\frac{1}{2} - \hat{x}_i) + B \\
&= \hat{x}_i + B - \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
Eu(x = x'''|\hat{x}_i \in A, \omega_1 = 0) &= -|x''' - \hat{x}_i| + B + \rho[-(\hat{x}_i - x') + B] \\
&= -(1 - \hat{x}_i) + B - \rho(\hat{x}_i - 0) + \rho B \\
&= (1-\rho)\hat{x}_i - 1 + (1+\rho)B
\end{aligned}$$

Note in the final calculation, when the politician chooses $x_2 = -\omega_1$, the assumption is that voter still re-elects the politician if the state of the world changes. I show even with this assumption, the politician does not deviate from choosing $x_2 = \omega_1$.

To find which types in group A choose $x = x'$, I solve the following inequality:

$$\begin{aligned}
Eu(x = x'|\hat{x}_t \in A) &\geq Eu(x = x''|\hat{x}_i \in A) \\
-(2-\rho)\hat{x}_i + (2-\rho)B &\geq \hat{x}_i + B - \frac{1}{2} \\
(-2+\rho-1)\hat{x}_i &\geq -\frac{1}{2} + (1-2+\rho)B \\
\hat{x}_i &\leq \frac{-\frac{1}{2} + (-1+\rho)B}{-3+\rho} \\
\hat{x}_i &\leq \frac{1 + 2(1-\rho)B}{2(3-\rho)}
\end{aligned}$$

As ρ approaches zero, the right hand side of the inequality approaches $\frac{1+2B}{6}$ from the left. As B is strictly greater than $\frac{1}{4}$, we then know that the right hand side of the inequality is strictly greater than $\frac{1}{4}$. As all politician in Group A have ideal points less than or equal to $\frac{1}{4}$, this equality holds for all politicians in Group A. Since no politicians from this group are choosing $x = x''$ and they do not choose $x = x'''$ over $x = x'$, there is no need to check any other inequalities. The probability of the state of the world changing is sufficiently small. Even if the voter is willing to re-elect a politician who chose $x_1 = \omega_1$ and $x_2 = -\omega_1$ if $\omega_2 = -\omega_1$, the politician does not deviate.

Voter Strategies:

Equilibrium calls for the voter to re-elect the first period incumbent as long as she choose the correct state of the world in the second period. If the incumbent, M, chooses $x_2 = \frac{1}{2}$ or $x_2 = -\omega_2$, the voter elects a random challenger, P, to hold office in the third period instead.

The voter expects a politician who choose $x_1 = \omega_1$ to only choose $x_2 = \omega_1$. Any other behavior is off the equilibrium path. I assign the voter's off the path beliefs such that if the voter observes $x_2 = \frac{1}{2}$, the voter believes the politician's ideal point is $\frac{1}{2}$ and if the voter observes $x_2 = -\omega_1$, the voter believes the politician's ideal point is $-\omega_1$. The former assignment would cause the voter to elect a random challenger, P, for the third period. The latter assignment would cause the voter to re-elect the incumbent, M, if the state of the world changes. As shown earlier, the politician still does not deviate.

Without loss of generality, assume the state of the world in the first period was zero, $\omega_1 = 0$.

When the state of the world remains the same:

$$\begin{aligned}
Eu_v(M|x_2 = 0, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &= u_v(0|0) \\
&= -\rho \\
Eu_v(M|x_2 = \frac{1}{2}, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &= u_v(\frac{1}{2}|0) \\
&= -\frac{1}{2} \\
Eu_v(M|x_2 = 1, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &= u_v(1|0) \\
&= -1 + \rho
\end{aligned}$$

When the state of the world changes:

$$\begin{aligned} Eu_v(M|x_2 = 0, x_1 = 0, \omega_2 = 1, \omega_1 = 0) &= u_v(0|1) \\ &= -1 + \rho \\ Eu_v(M|x_2 = \frac{1}{2}, x_1 = 0, \omega_2 = 1, \omega_1 = 0) &= u_v(\frac{1}{2}|1) \\ &= -\frac{1}{2} \\ Eu_v(M|x_2 = 1, x_1 = 0, \omega_2 = 1, \omega_1 = 0) &= u_v(1|1) \\ &= -\rho \end{aligned}$$

I start by comparing equilibrium path actions, where the politician M only chooses $x_2 = \omega_1$. If the state of the world remains the same, the voter should re-elect the politician. If the state of the world changes, the voter should elect a random challenger, P, for the third period.

$$\begin{aligned} Eu_v(M|x_2 = 0, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &\geq Eu_v(P) \\ -\rho &\geq -\frac{1}{2} \end{aligned}$$

As shown previously, this inequality holds strictly. If the politician chooses the correct state of the world, it is optimal for the voter to re-elect her.

$$\begin{aligned} Eu_v(M|x_2 = 1, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &\leq Eu_v(P) \\ -1 - \rho &\leq -\frac{1}{2} \end{aligned}$$

Again, it has been noted this inequality holds strictly. If the politician chooses the opposite state of the world, the voter's best response is to elect a random challenger.

I now check the voter's behavior given off the path beliefs. According to the off the path beliefs, the voter should elect a random challenger if the state of the world remains the same and the second period policy does not match it. If the state of the world changes, the voter should re-elect the politician if the state of the world is matched.

$$\begin{aligned} Eu_v(M|x_2 = \frac{1}{2}, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &\leq Eu_v(P) \\ -\frac{1}{2} &\leq -\frac{1}{2} \end{aligned}$$

Clearly, the voter is indifferent between re-electing the first period incumbent and electing the random challenger; therefore, the voter elects the random challenger.

$$
\begin{aligned}
Eu_v(M|x_2 = 1, x_1 = 0, \omega_2 = 0, \omega_1 = 0) &\leq Eu_v(P) \\
-1 + \rho &\leq -\frac{1}{2}
\end{aligned}
$$

The inequality holds strictly. If the politician does not match the state of the world, the voter removes the politician from office.

$$
\begin{aligned}
Eu_v(M|x_2 = \frac{1}{2}, x_1 = 0, \omega_2 = 1, \omega_1 = 0) &\leq Eu_v(P) \\
-\frac{1}{2} &\leq -\frac{1}{2}
\end{aligned}
$$

The voter is indifferent and elects the random challenger, P. Whenever the voter witnesses a politician who chose $x_1 = \omega_1$ choose $x_2 = \frac{1}{2}$, the voter elects a random challenger to the third period.

$$
\begin{aligned}
Eu_v(M|x_2 = 1, x_1 = 0, \omega_2 = 1, \omega_1 = 0) &\geq Eu_v(P) \\
-\rho &\geq -\frac{1}{2}
\end{aligned}
$$

This inequality holds strictly; therefore, whenever a first period incumbent chooses the correct state of the world in the second period, even if the second period policy is not the same as the first period policy, the voter re-elects the first period incumbent.

I now summarize formally what has been shown to happen in equilibrium when a first period incumbent who chose the correct state of the world is elected to office in the second period.

Politicians:

$$
\begin{gathered}
\sigma_2^{*M}(\hat{x}_i \in [0, \alpha_1] | x_1 = 0, \omega_1 = 0) = (x_2 = 0) \\
\sigma_2^{*M}(\hat{x}_i \in [\beta_1, 1] | x_1 = 1, \omega_1 = 1) = (x_2 = 1) \\
\mu^*_{\omega_2 = \omega_1} = 1 - \rho \qquad \mu^*_{\omega_2 = -\omega_1} = -\rho
\end{gathered}
$$

Voter:

$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = -\omega_2 | M, \omega_2 = \omega_1) = (M, P, P)$$
$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = -\omega_2 | M, \omega_2 = -\omega_1) = (M, P, P)$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0 | M, x_1 = 0, \omega_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2} | M, x_1 = 0, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1 | M, x_1 = 0, \omega_1 = 0) = 1$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0 | M, x_1 = 1, \omega_1 = 1) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2} | M, x_1 = 1, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1 | M, x_1 = 1, \omega_1 = 1) \sim U[\beta_1, 1]$$

$x_1 = \frac{1}{2}$ In equilibrium, the voter re-elects a first period incumbent who chooses $x_1 = \frac{1}{2}$. As discussed at the beginning of the previous subsection, the voter believes α_1 to be less than $\frac{1}{4}$ and β_1 to be greater than $\frac{3}{4}$. For now I assume this is true. Thus, the voter places some positive probability on a first period incumbent who chose $x_1 = \frac{1}{2}$ choosing each policy for the second period.

A politician's second period choice depends on her ideal point and the likelihood of re-election. The politician in the second period knows she will be re-elected as long as her policy choices matches the state of the world in the second period. There are politicians from each group choosing $x_1 = \frac{1}{2}$, so I analyzing second period behavior by group.

Politician Strategies:

Politicians with ideal points in the range $\left[\alpha_1, \frac{1}{4}\right]$ all belong to group A (see table **??**). It has already been shown all politicians in group A choose $x_2 = 0$ in the second period (see analysis resulting in equation 1).

Politicians with ideal points in the range $\left(\frac{1}{4}, \frac{1}{2}\right)$ are in group B.

$$\begin{aligned} Eu(x = x' | \hat{x}_t \in B) &= -|x' - \hat{x}_t| + B + 0 \\ &= -(\frac{1}{2} - \hat{x}_i) + B \\ &= \hat{x}_i + B - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu(x = x''|\hat{x}_i \in B) &= -|x'' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B] \\ &= -(\hat{x}_t - 0) + B + (1-\rho)[-(\frac{1}{2} - \hat{x}_i) + B] \\ &= -\rho\hat{x}_i - (1-\rho)\frac{1}{2} + (2-\rho)B \end{aligned}$$

$$\begin{aligned} Eu(x = x'''|\hat{x}_i \in B) &= -|x''' - \hat{x}_i| + B + \rho[-|x' - \hat{x}_i| + B] \\ &= -(1 - \hat{x}_i) + B - \rho(\frac{1}{2} - \hat{x}_i) + \rho B \\ &= (1+\rho)\hat{x}_i - 1 - \rho\frac{1}{2} + (1+\rho)B \end{aligned}$$

Now I check when politicians in group B compromise and choose x'' instead of x'.

$$\begin{aligned} Eu(x = x'|\hat{x}_t \in B) &\geq Eu(x = x''|\hat{x}_i \in B) \\ \hat{x}_i - \frac{1}{2} + B &\geq -\rho\hat{x}_i - (1-\rho)\frac{1}{2} + (2-\rho)B \\ (1-\rho)\hat{x}_i &\geq (-1+\rho+1)\frac{1}{2} + (2-\rho-1)B \\ \hat{x}_i &\geq \frac{(\rho)\frac{1}{2} + (1-\rho)B}{1+\rho} \\ \hat{x}_i &\geq \frac{\rho + 2(1-\rho)B}{2(1+\rho)} \end{aligned} \tag{7}$$

As ρ approaches zero, the right hand side of the inequality approaches B from the left. Any politician in Group B whose ideal point is less than B compromises and chooses $x = x''$ instead of of $x = x'$. Now I check to see if any politicians from this group compromises further and choose $x = x'''$.

$$\begin{aligned} Eu(x = x''|\hat{x}_i \in B) &\geq Eu(x = x'''|\hat{x}_i \in B) \\ -\rho\hat{x}_i + (2-\rho)B - \frac{1}{2} - \rho\frac{1}{2} &\geq (1+\rho)\hat{x}_i + (1+\rho)B - 1 - \rho\frac{1}{2} \\ (-\rho - 1 - \rho)\hat{x}_i &\geq (1 + \rho - 2 + \rho)B - \frac{1}{2} \\ -(1+2\rho)\hat{x}_i &\geq (-1+2\rho)B - \frac{1}{2} \\ \hat{x}_i &\leq \frac{(1-2\rho)B + \frac{1}{2}}{1-2\rho} \end{aligned}$$

As ρ approaches 0, the right hand side of the inequality approaches $B+\frac{1}{2}$ from the right. Since all politicians in Group B have ideal point to the left of $\frac{1}{2}$, this inequality is always true. Furthermore, no politician chooses $x = x'''$ over $x = x'''$; therefore, no politician in group B chooses $x = x'''$.

Let $\alpha_{2,0} = \dfrac{\rho + 2(1-\rho)B}{2(1+\rho)}$, from equation 5. All first period incumbents who choose $x_1 = \frac{1}{2}$ with ideal points to the right of $\alpha_{2,0}$ choose $x = 0$. It is worth noting that both the cutoff for politicians who choose $x_2 = 0$ is the same in for both the incumbent and the random challenger. This is expected as regardless of whether or not a politician in group A or B might get re-elected choosing $x = 1$, they never choose it.

Politicians with ideal points in the range $\left(\frac{1}{2}, \frac{3}{4}\right)$ are in group C.

$$
\begin{aligned}
Eu(x = x' | \hat{x}_t \in C) &= -|x' - \hat{x}_t| + B + 0 \\
&= -(\hat{x}_i - \frac{1}{2}) + B \\
&= -\hat{x}_i + B + \frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
Eu(x = x'' | \hat{x}_i \in C) &= -|x'' - \hat{x}_i| + B + \rho[-|x' - \hat{x}_i| + B] \\
&= -(1 - \hat{x}_i) + B - \rho(\hat{x}_i - \frac{1}{2}) + \rho B \\
&= (1-\rho)\hat{x}_i + (1+\rho)B - 1 + \rho\frac{1}{2}
\end{aligned}
$$

$$
\begin{aligned}
Eu(x = x''' | \hat{x}_i \in C) &= -|x''' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B] \\
&= -(\hat{x}_t - 0) + B + (1-\rho)[-(\hat{x}_i - \frac{1}{2}) + B] \\
&= -(2-\rho)\hat{x}_i + (2-\rho)B - (1-\rho)\frac{1}{2}
\end{aligned}
$$

Again I follow the same pattern of checking for politicians willing to compromise.

$$
\begin{aligned}
Eu(x = x'|\hat{x}_t \in C) &\geq Eu(x = x''|\hat{x}_i \in C) \\
-\hat{x}_i + B + \frac{1}{2} &\geq (1-\rho)\hat{x}_i + (1+\rho)B - 1 + \rho\frac{1}{2} \\
-(1+1-\rho)\hat{x}_i &\geq (1+\rho-1)B - 1 - \frac{1}{2} + \frac{1}{2}\rho \\
\hat{x}_i &\leq \frac{\rho B - \frac{3}{2} + \frac{1}{2}\rho}{-(2-\rho)} \\
\hat{x}_i &\leq \frac{3 - 2\rho B - \rho}{2(2-\rho)} \qquad (8)
\end{aligned}
$$

A politician in group C chooses $x = x'$ whenever the above inequality is satisfied. As ρ approaches 0, the right hand side approaches $\frac{3}{4}$ from the left, which includes all politicians in group C. It is assumed ρ is strictly positive, but sufficiently close to zero. As the limit of the right hand side of the inequality is approached from the left as ρ approaches zero, there are some politicians who are compromising.

Now I see if any of those politicians compromise even further.

$$
\begin{aligned}
Eu(x = x''|\hat{x}_i \in C) &\geq Eu(x = x'''|\hat{x}_i \in C) \\
(1-\rho)\hat{x}_i + (1+\rho)B - 1 + \rho\frac{1}{2} &\geq -(2-\rho)\hat{x}_i + (2-\rho)B - \frac{1}{2} + \frac{1}{2}\rho \\
(1-\rho+2-\rho)\hat{x}_i &\geq (2-\rho-1-\rho)B - \frac{1}{2} + 1 \\
\hat{x}_i &\geq \frac{B + \frac{1}{2}}{3-2\rho} \\
\hat{x}_i &\geq \frac{2B+1}{2(3-2\rho)}
\end{aligned}
$$

As ρ approaches 0, the inequality heavily relies on the office benefit B; however, B varies within $(\frac{1}{4}, \frac{1}{2})$, the right hand side varies within $(\frac{1}{4}, \frac{1}{3})$. Thus, the inequality is always strictly true and no politician from group C compromises by choosing her third preference.

Finally, politicians with ideal points in the range $\left[\frac{3}{4}, \beta_1\right]$ are all in group D.

$$
\begin{aligned}
Eu(x = x'|\hat{x}_i \in D) &= -|x' - \hat{x}_i| + B + \rho(-|x' - \hat{x}_i| + B) \\
&= -(1-\hat{x}_i) + B - \rho(1-\hat{x}_i) + \rho B \\
&= (1+\rho)\hat{x}_i + (1+\rho)B - 1 - \rho
\end{aligned}
$$

$$\begin{aligned} Eu(x = x''|idp \in D) &= -|x'' - \hat{x}_i| + B + 0 \\ &= -(\hat{x}_i - \frac{1}{2}) + B \\ &= -\hat{x}_i + B - \frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu(x = x'''|\hat{x}_i \in D) &= -|x''' - \hat{x}_i| + B + (1-\rho)(-|x' - \hat{x}_i| + B) \\ &= -(\hat{x}_i - 0) + B - (1-\rho)(1 - \hat{x}_i) + (1-\rho)B \\ &= -\rho\hat{x}_i + (2-\rho)B - 1 + \rho \end{aligned}$$

I begin by checking which members of group D compromise to their second choice, if any.

$$\begin{aligned} Eu(x = x'|\hat{x}_i \in D) &\geq Eu(x = x''|idp \in D) \\ (1+\rho)\hat{x}_i + (1+\rho)B - 1 - \rho &\geq -\hat{x}_i + B - \frac{1}{2} \\ (1+\rho+1)\hat{x}_i &\geq (1-1-\rho)B - \frac{1}{2} + 1 + \rho \\ \hat{x}_i &\geq \frac{-\rho B + \frac{1}{2} + \rho}{2+\rho} \\ \hat{x}_i &\geq \frac{1 - 2\rho B + \rho}{2(2+\rho)} \end{aligned}$$

As ρ approaches 0 in this case, the right hand side of the inequality approaches $\frac{1}{4}$ from the left and this inequality is strictly true, implying there are no politicians in group D who are willing to compromise. Since no politician from group D are choosing their second preference, there is no need to check whether anyone chooses their third preference.

As all politician from group D are choosing their first preference, I am now able to set $\beta_{2,0} = \dfrac{3 - 2\rho B - \rho}{2(2-\rho)}$ where $\beta_{2,0}$ is the cut-point such that any politician with an ideal point at or to the right of $\beta_{2,0}$ chooses $x = 1$.

Before moving on to check that the voter is acting optimally, I summarize formally the strategies of the politicians who chose $x_1 = \frac{1}{2}$.

$$\sigma_2^{*M}\left(\hat{x}_i \in (\alpha_1, \alpha_{2,0}], \hat{x}_i \in (\alpha_{2,0}, \beta_{2,0}), \hat{x}_i \in [\beta_{2,0}, \beta_1) | \omega_1 = 0\right) = \left(x_2 = 0, x_2 = \frac{1}{2}, x_2 = 1\right)$$

$$\alpha_{2,0} = \frac{\rho + 2(1-\rho)B}{2(1+\rho)} \qquad \beta_{2,0} = \frac{3 - 2\rho B - \rho}{2(2-\rho)}$$

$$\sigma_2^{*M}\left(\hat{x}_i \in (\alpha_1, \alpha_{2,1}], \hat{x}_i \in (\alpha_{2,1}, \beta_{2,1}), \hat{x}_i \in [\beta_{2,1}, \beta_1) | \omega_1 = 1\right) = \left(x_2 = 0, x_2 = \frac{1}{2}, x_2 = 1\right)$$

$$\alpha_{2,1} = 1 - \beta_{2,0} \qquad \beta_{2,1} = 1 - \alpha_{2,0}$$

Voter Strategies:

In equilibrium, the voter re-elects the first period incumbent, N, as long as the second period policy matches the state of the world in the second period. Without loss of generality, assume the state of the world in the first period was zero, $\omega_1 = 0$.

When the state of the world remains the same:

$$\begin{aligned} Eu_v(M|x_2 = 0, x_1 = \frac{1}{2}, \omega_2 = 0, \omega_1 = 0) &= \frac{1}{\alpha_{2,0} - \alpha_1}\left[\left(\frac{1}{4} - \alpha_1\right) u_v(0|0) + \left(\alpha_{2,0} - \frac{1}{4}\right) u_v(\frac{1}{2}|0)\right] \\ &= \frac{1}{\alpha_{2,0} - \alpha_1}\left[\rho\alpha_1 + \alpha_{2,0}\left(\frac{1}{8} - \frac{1}{4}\rho\right) - \frac{1}{2}\right] \end{aligned}$$

$$\begin{aligned} Eu_v(M|x_2 = \frac{1}{2}, x_1 = \frac{1}{2}, \omega_2 = 0, \omega_1 = 0) &= u_v(\frac{1}{2}|0) \\ &= -\frac{1}{2} \\ Eu_v(M|x_2 = 1, x_1 = \frac{1}{2}, \omega_2 = 0, \omega_1 = 0) &= \frac{1}{\beta_1 - \beta_{2,0}}\left[\left(\frac{3}{4} - \beta_{2,0}\right) u_v(\frac{1}{2}|0) + \left(\beta_1 - \frac{3}{4}\right) u_v(1|0)\right] \\ &= \frac{1}{\beta_1 - \beta_{2,0}}\left[\frac{1}{2}\beta_{2,0} + (-1+\rho)\beta_1 + \frac{3}{8} - \frac{3}{4}\rho\right] \end{aligned}$$

When the state of the world changes:

$$
\begin{aligned}
Eu_v(M|x_2=0,x_1=\frac{1}{2},\omega_2=1,\omega_1=0) &= \frac{1}{\alpha_{2,0}-\alpha_1}\left[\left(\frac{1}{4}-\alpha_1\right)u_v(0|1)+\left(\alpha_{2,0}-\frac{1}{4}\right)u_v(\frac{1}{2}|1)\right]\\
&= \frac{1}{\alpha_{2,0}-\alpha_1}\left[(-1+\rho)\alpha_1-\frac{1}{2}\alpha_2-\frac{1}{8}+\frac{1}{4}\rho\right]\\
Eu_v(M|x_2=\frac{1}{2},x_1=\frac{1}{2},\omega_2=1,\omega_1=0) &= u_v(\frac{1}{2}|1)\\
&= -\frac{1}{2}\\
Eu_v(M|x_2=1,x_1=\frac{1}{2},\omega_2=1,\omega_1=0) &= \frac{1}{\beta_1-\beta_{2,0}}\left[\left(\frac{3}{4}-\beta_{2,0}\right)u_v(\frac{1}{2}|1)+\left(\beta_1-\frac{3}{4}\right)u_v(1|1)\right]\\
&= \frac{1}{\beta_1-\beta_{2,0}}\left[\frac{1}{2}\beta_{2,0}-\frac{3}{8}+\frac{3}{4}\rho-\rho\beta_1\right]
\end{aligned}
$$

At this points α_1 and β_1 have not been solved for, but knowing the strategies the voter plays in equilibrium allows me to solve for limits on the first period cut-points.

To reiterate, in equilibrium, the voter re-elects the first period incumbent, M, as long as the second period policy matches the state of the world.

When the state of the world remains the same:

$$
\begin{aligned}
Eu_v(M|x_2=0,x_1=\frac{1}{2},\omega_2=0,\omega_1=0) &\geq Eu_v(P)\\
\frac{1}{\alpha_{2,0}-\alpha_1}\left[\rho\alpha_1+\alpha_{2,0}\left(\frac{1}{8}-\frac{1}{4}\rho\right)-\frac{1}{2}\right] &\geq -\frac{1}{2}\\
\frac{(2(1-\rho)B+\rho)(5-2\rho)}{16(1+\rho)(\frac{1}{2}-\rho)} &\geq \alpha_1
\end{aligned}
$$

When the state of the world changes:

$$
\begin{aligned}
Eu_v(M|x_2=1,x_1=\frac{1}{2},\omega_2=1,\omega_1=0) &\geq Eu_v(P)\\
\frac{1}{\beta_1-\beta_{2,0}}\left[\frac{1}{2}\beta_{2,0}-\frac{3}{8}+\frac{3}{4}\rho-\rho\beta_1\right] &\geq -\frac{1}{2}\\
\frac{3-6\rho}{4(1-2\rho)} &\leq \beta_1
\end{aligned}
$$

As ρ approaches zero, the upper bound of α_1 approaches $\frac{5}{4}B$ from the right. The lower bound of the office benefit is $\frac{1}{4}$, meaning the lowest upper bound of α_1 is $\frac{5}{16}$, or just over $\frac{1}{4}$. Under the assumptions in place, α_1

is strictly less than $\frac{1}{4}$, so the inequality always holds. If the state of the world remains the same and the first period incumbent chooses the correct state of the world in the second period, the voter is best off re-electing the first period incumbent.

For the second inequality, as ρ approaches zero, the left hand side of the inequality approaches $\frac{3}{4}$ from the left. Under the current assumptions, β_1 is strictly greater than $\frac{3}{4}$ and thus the inequality holds strictly. If the state of the world changes and the first period incumbent chooses the correct state of the world, the voter is best off re-electing the first period incumbent.

Now I check that the voter does not re-elect the first period incumbent if the politician chooses $x_2 = \frac{1}{2}$ or the opposite state of the world.

When the state of the world remins the same:

$$\begin{aligned} Eu_v(M|x_2 = 1, x_1 = \frac{1}{2}, \omega_2 = 0, \omega_1 = 0) &\leq Eu_v(P) \\ -\frac{1}{2} &\leq -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu_v(M|x_2 = 1, x_1 = \frac{1}{2}, \omega_2 = 0, \omega_1 = 0) &\leq Eu_v(P) \\ \frac{1}{\beta_1 - \beta_{2,0}}\left[\frac{1}{2}\beta_{2,0} + (-1+\rho)\beta_1 + \frac{3}{8} - \frac{3}{4}\rho\right] &\leq -\frac{1}{2} \\ \frac{3-6\rho}{4(1-2\rho)} &\leq \beta_1 \end{aligned}$$

When the state of the world changes:

$$\begin{aligned} Eu_v(M|x_2 = \frac{1}{2}, x_1 = \frac{1}{2}, \omega_2 = 1, \omega_1 = 0) &\leq Eu_v(P) \\ -\frac{1}{2} &\leq -\frac{1}{2} \end{aligned}$$

$$\begin{aligned} Eu_v(M|x_2 = 0, x_1 = \frac{1}{2}, \omega_2 = 1, \omega_1 = 0) &\leq Eu_v(P) \\ \frac{1}{\alpha_{2,0} - \alpha_1}\left[(-1+\rho)\alpha_1 - \frac{1}{2}\alpha_2 - \frac{1}{8} + \frac{1}{4}\rho\right] &\leq -\frac{1}{2} \\ \frac{1-\rho}{4(1-\rho)} &\geq \alpha_1 \end{aligned}$$

Whenever the politician chooses $x_2 = \frac{1}{2}$, the voter is indifferent and re-elects the random challenger in equilibrium.

It has already been shown under the assumptions, the limit placed on the lower bound of β_1 holds strictly. When the state of the world remains the same and the incumbent chooses the wrong state of the world, the voter is better off electing a random challenger.

Finally, as ρ approaches zero, the limit on the upper bound of α_1 approaches $\frac{1}{4}$ from the right. Under the equilibrium assumption that α_1 is strictly less than $\frac{1}{4}$, the inequality holds strictly. If the state of the world changes and the incumbent chooses the wrong state of the world, the voter is best off electing a random challenger, P, for the third period.

I now summarize formally the politician and voter strategies in the second period when $x_1 = \frac{1}{2}$.

Politician:

$$\sigma_2^{*M}\left(\hat{x}_i \in (\alpha_1, \alpha_{2,0}], \hat{x}_i \in (\alpha_{2,0}, \beta_{2,0}), \hat{x}_i \in [\beta_{2,0}, \beta_1) | x_1 = \frac{1}{2}, \omega_1 = 0\right) = \left(x_2 = 0, x_2 = \frac{1}{2}, x_2 = 1\right)$$

$$\alpha_{2,0} = \frac{\rho + 2(1-\rho)B}{2(1+\rho)} \qquad \beta_{2,0} = \frac{3 - 2\rho B - \rho}{2(2-\rho)}$$

$$\sigma_2^{*M}\left(\hat{x}_i \in (\alpha_1, \alpha_{2,1}], \hat{x}_i \in (\alpha_{2,1}, \beta_{2,1}), \hat{x}_i \in [\beta_{2,1}, \beta_1) | x_1 = \frac{1}{2}, \omega_1 = 1\right) = \left(x_2 = 0, x_2 = \frac{1}{2}, x_2 = 1\right)$$

$$\alpha_{2,1} = 1 - \beta_{2,0} \qquad \beta_{2,1} = 1 - \alpha_{2,0}$$

$$\mu^*_{\omega_2 = \omega_1} = 1 - \rho \qquad \mu^*_{\omega_2 = -\omega_1}$$

Voter:

$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = -\omega_2 | M, x_1 = \frac{1}{2}, \omega_2 = \omega_1) = (M, P, P)$$

$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = 1\omega_2 | M, x_1 = \frac{1}{2}, \omega_2 = -\omega_1) = (M, P, P)$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 0 | M, x_1 = \frac{1}{2}) \sim U[\alpha_1, \alpha_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2} | M, x_1 = \frac{1}{2}\right) \sim U\left(\alpha_{2,\omega_1}, \beta_{2,\omega_1}\right)$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 1 | M, x_1 = \frac{1}{2}) \sim U\left[\beta_{2,\omega_1}, \beta_1\right]$$

$x_1 = -\omega$ The voter re-electing a politician from the first period who does not match the state of the world, $x_1 = -\omega_1$, is an off the path action, but it is still possible to figure out optimal actions for the players. Without loss of generality, assume the state of the world in the first period is zero, $\omega_1 = 0$.

Politician Strategies:

Under the assumption β_1 is strictly greater than $\frac{3}{4}$, all politicians who choose $x_1 = 1$ when the state of the world is zero are in group D. In the previous section, it was shown all politicians in group D choose $x_2 = 1$, even though deviating to $x_2 = 0$ is more likely to allow re-election. As $x' = 1$ for politicians in group D, if elected to the third period, the politician chooses $x_3 = 1$.

A politician who chooses the opposite state of the world in the first period chooses the same policy in the second period.

Voter Strategies:

In equilibrium, the voter re-elects the first period incumbent, M, to hold office in the third period if the incumbent chooses the correct state of the world. I show this holds true even if the politician chose the opposite state of the world in the first period.

Assuming the state of the world in the first period was zero, the voter believes the politician only chooses $x_2 = 1$. Let the voter have analogous off the path beliefs as when the politician matches the state of the world in the first period. That is, if the voter observes $x_2 = \frac{1}{2}$, the voter believes the politician's ideal point is $\frac{1}{2}$ and if the voter observes $x_2 = \omega_1$, the voter believes the politician's ideal point is ω_1.

When the state of the world remains the same:

$$\begin{aligned} Eu_v(M|x_2 = 1, x_1 = 1, \omega_2 = 0, \omega_1 = 0) &= u_v(1|0) \\ &= -1 + \rho \\ Eu_v(M|x_2 = \frac{1}{2}, x_1 = 1, \omega_2 = 0, \omega_1 = 0) &= u_v(\frac{1}{2}|0) \\ &= -\frac{1}{2} \\ Eu_v(M|x_2 = 0, x_1 = 1, \omega_2 = 0, \omega_1 = 0) &= u_v(0|0) \\ &= -1\rho \end{aligned}$$

When the state of the world changes:

$$\begin{aligned} Eu_v(M|x_2 = 1, x_1 = 1, \omega_2 = 1, \omega_1 = 0) &= u_v(1|1) \\ &= -\rho \\ Eu_v(M|x_2 = \frac{1}{2}, x_1 = 1, \omega_2 = 1, \omega_1 = 0) &= u_v(\frac{1}{2}|1) \\ &= -\frac{1}{2} \\ Eu_v(M|x_2 = 0, x_1 = 1, \omega_2 = 1, \omega_1 = 0) &= u_v(0|1) \\ &= -1 + \rho \end{aligned}$$

Note the voter's expected utilities from re-election a first period incumbent who choose the wrong state of the world in the first period are analogous to the voter's expected utilities from re-election a first period incumbent who chose the correct state of the world in the first period. Therefore, it is still optimal for the voter to re-elect the incumbent if the state of the world is matched in the second period and elect a random challenger if the state is not matched.

I now summarize formally the strategies and beliefs of the actors when the first period incumbent is re-elected after choosing the opposite state of the world in the first period.

Politicians:

$$\sigma_2^{*M}(\hat{x}_i \in [\beta_1, 1] | x_1 = 1, \omega_1 = 0) = (x_2 = 1)$$
$$\sigma_2^{*M}(\hat{x}_i \in [0, \alpha_1] | x_1 = 0, \omega_1 = 1) = (x_2 = 0)$$
$$\mu^*_{\omega_2 = \omega_1} = 1 - \rho \qquad \mu^*_{\omega_2 = -\omega_1} = -\rho$$

Voter:

$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = -\omega_2 | M, \omega_2 = \omega_1) = (M, P, P)$$
$$\sigma_2^{*v}(x_2 = \omega_2, x_2 = \frac{1}{2}, x_2 = -\omega_2 | M, \omega_2 = -\omega_1) = (M, P, P)$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0 | M, x_1 = 0, \omega_1 = 1) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2} | M, x_1 = 0, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1 | M, x_1 = 0, \omega_1 = 1) = 1$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0 | M, x_1 = 1, \omega_1 = 0) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2} | M, x_1 = 1, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1 | M, x_1 = 1, \omega_1 = 0) \sim U\left[\beta_1, 1\right]$$

Before I summarize the results from period two, I want to make note of the strategies of the actors when the incumbent is re-elected. The voter uses the same strategy regardless of the action the politicians took in the first period. Only the beliefs of the voter change. The strategies of the politicians differ based on what they choose in the first period; however, this can be entirely attributed to the position of their ideal points. Regardless of first period actions, all politicians with ideal points to the left of α_{2,ω_1} choose $x_2 = 0$, all politicians to with ideal points to the right of β_{2,ω_1} choose $x_2 = 1$, and all politicians with ideal points in between choose $x_2 = \frac{1}{2}$. Instead of writing several separate strategies for the actors, I condense them.

When M is in office:

$$\sigma_2^{*M}(\omega_1) = \begin{cases} x_1 = 0 & \text{if } \hat{x}_i \in [0, \alpha_{2,\omega_1}] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha_{2,\omega_1}, \beta_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1 - \rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(M, \omega_2 = \omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(M, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases}$$

Beliefs following equilibrium actions:

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 0, \omega_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 0, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 0, \omega_1 = 0) = 1$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 1, \omega_1 = 1) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 1, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 1, \omega_1 = 1) \sim U\left[\beta_1, 1\right]$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = \frac{1}{2}) \sim U[\alpha_1, \alpha_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = \frac{1}{2}\right) \sim U\left(\alpha_{2,\omega_1}, \beta_{2,\omega_1}\right)$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = \frac{1}{2}) \sim U\left[\beta_{2,\omega_1}, \beta_1\right]$$

Beliefs following off-the-path actions:

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 0, \omega_1 = 1) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 0, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 0, \omega_1 = 1) = 1$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 1, \omega_1 = 0) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 1, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 1, \omega_1 = 0) \sim U\left[\beta_1, 1\right]$$

When N is in office:

$$\sigma_2^{*N}(\omega_1) \begin{cases} x_2 = 0 & \text{if } \hat{x}_i \in [0, \alpha'_{2,\omega_1}] \\ x_2 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta'_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1 - \rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(N, \omega_2 = \omega_1) = \begin{cases} N & \text{i f } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(N, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ M & \text{if } x_2 = \frac{1}{2} \\ M & \text{if } x_2 = -\omega_2 \end{cases}$$

$$\mu^*_{2,\hat{x}_i}(x_2|N) \sim U[0, \alpha'_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}(x_2|N) \sim U(\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \quad \mu^*_{2,\hat{x}_i}(x_2 = 1|N) \sim U[\beta'_{2,\omega_1}, 1]$$

$$\alpha'_{2,0} = \frac{2(1-\rho)B + \rho}{2 + 2\rho} \qquad \beta'_{2,0} = \frac{3}{4}$$

$$\alpha'_{2,1} = 1 - \beta'_{2,0} \qquad \beta'_{2,1} = 1 - \alpha'_{2,0}$$

B.3 First Period

At the start of the first period, the incumbent chosen by Nature believes each state of the world is equally likely. In equilibrium, the voter re-elects the incumbent so long as the incumbent chooses the correct state of the world or $x_1 = \frac{1}{2}$. Just as in the previous analysis of per period behavior, I begin with the actions of the politician.

Politician Strategies:

The ideal point of the politicians dictate their behavior in the second and third period. I determine the actions of politicians according to the group they belong to.

B.3.1 Group A

In the second period, all incumbent politicians from group A choose their ideal policy, regardless of the state of the world.

$$\begin{aligned}
Eu_A(x') =& - |x' - \hat{x}_i| + B + \frac{1}{2}\{-|x' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{0 + \rho[-|x' - \hat{x}_i| + B]\} \\
=& - (\hat{x}_i - 0) + B + \frac{1}{2}\{-(\hat{x}_i - 0) + B + (1-\rho)[-(\hat{x}_i - 0) + B]\} \\
&+ \frac{1}{2}\{\rho[-(\hat{x}_i - 0) + B]\} \\
=& - 2\hat{x}_i + 2B
\end{aligned}$$

$$\begin{aligned}
Eu_A(x'') =& - |x'' - \hat{x}_i| + B + \frac{1}{2}\{-|x' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{-|x' - \hat{x}_i| + B + \rho[-|x' - \hat{x}_i| + B]\} \\
=& - (\frac{1}{2} - \hat{x}_i) + B + \frac{1}{2}\{-(\hat{x}_i - 0) + B + (1-\rho)[-(\hat{x}_i - 0) + B]\} \\
&+ \frac{1}{2}\{-(\hat{x}_i - 0) + B + \rho[-(\hat{x}_i - 0) + B]\} \\
=& - \frac{1}{2}\hat{x}_i + \frac{5}{2}B - \frac{1}{2}
\end{aligned}$$

$$\begin{aligned}
Eu_A(x''') =& - |x''' - \hat{x}_i| + B + \frac{1}{2}\{0 + \rho[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{-|x' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B]\} \\
=& - (1 - \hat{x}_i) + B + \frac{1}{2}\{0 + \rho[-(\hat{x}_i - 0) + B]\} \\
&+ \frac{1}{2}\{-(\hat{x}_i - 0) + B + \rho[-(\hat{x}_i - 0) + B]\} \\
=& (\frac{1}{2} - \rho)\hat{x}_i + (\frac{3}{2} + \rho)B - 1
\end{aligned}$$

Now I can check who compromises.

$$\begin{aligned}
Eu_A(x') &\geq Eu_A(x'') \\
-2\hat{x}_i + 2B &\geq -\frac{1}{2}\hat{x}_i + \frac{5}{2}B - \frac{1}{2} \\
-\frac{3}{2}\hat{x}_i &\geq (\frac{5}{2} - 2)B - \frac{1}{2} \\
\hat{x}_i &\leq \frac{1-B}{3}
\end{aligned} \tag{9}$$

Based on the range of the office benefit B, the right hand side of the inequality varies within $\left(\frac{1}{6}, \frac{1}{4}\right)$; therefore, depending on the size of the office benefit, some politicians may compromise. The larger the office benefit is, the more likely the politician is to compromise.

Next, I check if any compromising politicians would compromise further.

$$\begin{aligned}
Eu_A(x'') &\geq Eu_A(x''') \\
-\frac{1}{2}\hat{x}_i + \frac{5}{2}B - \frac{1}{2} &\geq (\frac{1}{2} - \rho)\hat{x}_i + (\frac{3}{2} + \rho)B - 1 \\
(-\frac{1}{2} - \frac{1}{2} + \rho)\hat{x}_i &\geq (\frac{3}{2} + \rho - \frac{5}{2})B - 1 + \frac{1}{2} \\
\hat{x}_i &\leq \frac{(-1+\rho)B - \frac{1}{2}}{-1+\rho} \\
\hat{x}_i &\leq \frac{2 + 2(1-\rho)B}{2-2\rho}
\end{aligned}$$

As ρ approaches 0, the right hand side of the inequality approaches to $B + \frac{1}{2}$ from the right. This inequality holds strictly. No politician in group A chooses $x = x'''$.

B.3.2 Group D

$$\begin{aligned}
Eu_D(x') =& -|x' - \hat{x}_i| + B + \frac{1}{2}\{0 + B + \rho[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{-|x' - \hat{x}_i| + (1-\rho)[-|x' - \hat{x}_i| + B]\} \\
=& -(1 - \hat{x}_i) + B + \frac{1}{2}\{\rho[-(1-\hat{x}_i) + B]\} \\
&+ \frac{1}{2}\{-(1-\hat{x}_i) + B + (1-\rho)[-(1-\hat{x}_i) + B]\} \\
=& 2\hat{x}_i + 2B - 2
\end{aligned}$$

$$\begin{aligned}
Eu_D(x'') =& - |x'' - \hat{x}_i| + B + \frac{1}{2}\{-|x' - \hat{x}_i| + B + \rho[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{-|x' - \hat{x}_i| + B + (1-\rho)[-|x' - \hat{x}_i| + B]\} \\
=& - (\hat{x}_i - \frac{1}{2}) + B + \frac{1}{2}\{-(1-\hat{x}_i) + B + \rho[-(1-\hat{x}_i) + B]\} \\
&+ \frac{1}{2}\{-(1-\hat{x}_i) + B + (1-\rho)[-(1-\hat{x}_i) + B]\} \\
=& \frac{1}{2}\hat{x}_i + \frac{5}{2}B - 1
\end{aligned}$$

$$\begin{aligned}
Eu_D(x''') =& - |x''' - \hat{x}_i| + B + \frac{1}{2}\{-|x' - \hat{x}_i| + B + \rho[-|x' - \hat{x}_i| + B]\} \\
&+ \frac{1}{2}\{0 + \rho[-|x' - \hat{x}_i| + B]\} \\
=& - (\hat{x}_i - 0) + B + \frac{1}{2}\{-(1-\hat{x}_i) + B + \rho[-(1-\hat{x}_i) + B]\} \\
&+ \frac{1}{2}\{0 + \rho[-(1-\hat{x}_i) + B]\} \\
=& (-\frac{1}{2} + \rho)\hat{x}_i + (\frac{3}{2} + \rho)B - \frac{1}{2} - \rho
\end{aligned}$$

The next step is to check for compromises.

$$\begin{aligned}
Eu_D(x') &\geq Eu_D(x'') \\
2\hat{x}_i + 2B - 2 &\geq \frac{1}{2}\hat{x}_i + \frac{5}{2}B - 1 \\
\left(2 - \frac{1}{2}\right)\hat{x}_i &\geq \left(\frac{5}{2} - 2\right)B - 1 + 2 \\
\frac{3}{2}\hat{x}_i &\geq \frac{1}{2}B + 1 \\
\hat{x}_i &\geq \frac{B+2}{3} \qquad (10)
\end{aligned}$$

Politicians with ideal points at and to the right of $\frac{B+2}{3}$ choose $x = x'$, while politicians to the left compromise. I now check if any compromising politicians compromise further.

$$
\begin{aligned}
Eu_D(x'') &\geq Eu_D(x''') \\
\frac{1}{2}\hat{x}_i + \frac{5}{2}B - 1 &\geq (-\frac{1}{2} + \rho)\hat{x}_i + (\frac{3}{2} + \rho)B - \frac{1}{2} - \rho \\
\left(\frac{1}{2} + \frac{1}{2} - \rho\right)\hat{x}_i &\geq \left(\frac{3}{2} - \frac{5}{2} + \rho\right)B - \frac{1}{2} + 1 - \rho \\
\hat{x}_i &\geq \frac{(-1+\rho)B + \frac{1}{2} - \rho}{1-\rho} \\
\hat{x}_i &\geq \frac{1 - 2(1-\rho)B - 2\rho}{2(1-\rho)}
\end{aligned}
$$

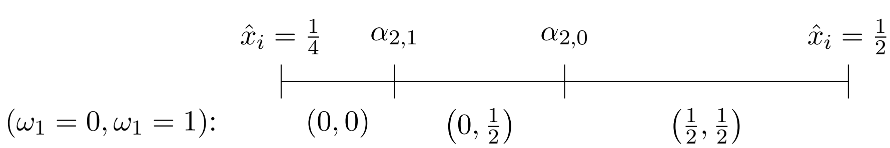
As ρ approaches 0, the right hand side of the inequality approaches $\frac{1-2B}{2}$ from the left. The ideal points of politicians in group D are all greater than this. The inequality holds strictly and no politician in group D chooses $x = x'''$.

The cut-points α_1 and β_1 are less than $\frac{1}{4}$ and greater than $\frac{3}{4}$ respectively. The analysis for a first period incumbent re-elected to the second period holds.

B.3.3 Group B

Both groups B and C must be subdivided into three different groups based on their actions in the second period, as shown in figure 8. A politician's actions depend on whether her ideal point is in the interval $[\frac{1}{4}, \alpha_{2,1}]$, $(\alpha_{2,1}, \alpha_{2,0}]$, or $(\alpha_{2,0}, \frac{1}{2}]$. The order of the cutoffs is determined by analyzing both as ρ approaches zero. Then we know $\alpha_{2,1}$ is very close to $\frac{1}{4}$ and $\alpha_{2,0}$ is very close to B, which is always strictly greater than $\frac{1}{2}$.

Figure 8: *The interval is comprised of all the ideal points of politicians in group B. The parenthesis under the three segments indicate the second period policy choices of politicians whose ideal points are contained in those segments when the state of the world in the first period turns out to be zero or one respectively.*



Depending on the value of the office benefit, it is possible for the order of $\alpha_{2,1}$ and $\alpha_{2,0}$ in figure 8 to be reversed.

Regardless of the ordering or the cutoffs, the sequential actions taken by each politician are based on their ideal points.

In order to show all the politician's in group B choose $x = x' = \frac{1}{2}$, let $u_2(x)$ and $u_3(x)$ be the expected utility from choosing policy x in the second and third period respectively. In order to demonstrate choosing

$x = x'$ is the best strategy for all politicians in group B, I examine the extreme case when $\hat{x}_i = \frac{1}{4}$, which is the lower limit of ideal points for politicians in group B. Choosing $x_1 = \frac{1}{2}$ is most costly for a politician when $\hat{x}_i = \frac{1}{4}$; therefore, if this politician prefers choosing $x = \frac{1}{2}$ over compromising to $x = x'' = 0$, then all politicians in group B choose $x = x' = \frac{1}{2}$.

Regardless of the order of $\alpha_{2,1}$ and $\alpha_{2,0}$, a politician with the ideal point $\hat{x}_i = \frac{1}{4}$ always chooses $x = 0$ in the second period, if elected. I then calculate the expected utility of the politician from choosing x', x'', or x'''.

$$
\begin{aligned}
Eu_B(x') =& -|x' - \hat{x}_i| + B + \frac{1}{2}\left[u_2(0) + (1-\rho)u_3(\frac{1}{2}) + \rho * 0\right] \\
&+ \frac{1}{2}\left[u_2(0) + (1-\rho) * 0 + \rho u_3(\frac{1}{2})\right] \\
=& -\left(\frac{1}{2} - \frac{1}{4}\right) + B + u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right) \\
=& -\frac{1}{4} + B + u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
Eu_B(x'') =& -|x'' - \hat{x}_i| + B + \frac{1}{2}\left[u_2(0) + (1-\rho)u_3\left(\frac{1}{2}\right)\right] \\
&+ \frac{1}{2}\left[0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
=& -\left(\frac{1}{4} - 0\right) + B + \frac{1}{2}u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right) \\
=& -\frac{1}{4} + B + \frac{1}{2}u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
Eu_B(x''') =& -|x''' - \hat{x}_i| + B + \frac{1}{2}\left[0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
&+ \frac{1}{2}\left[u_2(0) + (1-\rho) * 0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
=& -\left(1 - \frac{1}{4}\right) + B + \frac{1}{2}u_2(0) + \rho u_3\left(\frac{1}{2}\right) \\
=& -\frac{3}{4} + \frac{1}{2}u_2(0) + \rho u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

Now I can compare the payoffs of the different strategies.

$$
\begin{gathered}
Eu_B(x') \geq Eu_B(x'') \\
-\frac{1}{4} + B + u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right) \geq -\frac{1}{4} + B + \frac{1}{2}u_2(0) + \frac{1}{2}u_3\left(\frac{1}{2}\right) \\
u_2(0) \geq \frac{1}{2}u_2(0)
\end{gathered}
$$

The inequality always holds, meaning the politician never compromises to her second choice and by extension never compromise to her third choices either. A politician in group B with the ideal point $\hat{x}_i = \frac{1}{4}$ always chooses $x = x' = \frac{1}{2}$; therefore, all politicians in group B choose $x = x' = \frac{1}{2}$.

Now I know the first period cut point delimiting the politicians who choose $x_1 = \frac{1}{2}$ from those who choose $x_1 = 0$ is $\alpha_1 = \frac{1-B}{3}$.

B.3.4 Group C

In order to determine the actions of politicians in group C, I follow an analogous procedure to the one followed for group B. In this case, a politician with the ideal point $\hat{x}_i = \frac{3}{4}$ finds choosing $x = x' = \frac{1}{2}$ the most costly and $x = x'' = 1$ the least costly. If a politician with the ideal point $\hat{x}_i = \frac{3}{4}$ chooses $x = x' = \frac{1}{2}$, then all politicians in group C choose $x = \frac{1}{2}$.

The next step is to calculate the expected utility of each policy choice for the politician. Again let $u_2(x)$ and $u_3(x)$ be the expected utility from choosing policy x in periods two and three respectively.

$$
\begin{aligned}
Eu_C(x') = &- |x' - \hat{x}_i| + B + \frac{1}{2}\left[u_2\left(1\right) + (1-\rho) * 0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
&+ \frac{1}{2}\left[u_2(1) + (1-\rho) u_3\left(\frac{1}{2}\right) + \rho * 0\right] \\
= &- \left(\frac{3}{4} - \frac{1}{2}\right) + B + u_2(1) + \frac{1}{2} u_3\left(\frac{1}{2}\right) \\
= &- \frac{1}{4} + B + u_2(1) + \frac{1}{2} u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
Eu_C(x'') = &- |x'' - \hat{x}_i| + B + \frac{1}{2}\left[0 + (1-\rho) * 0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
&+ \frac{1}{2}\left[u_2(1) + (1-\rho) u_3\left(\frac{1}{2}\right) + \rho * 0\right] \\
= &- \left(1 - \frac{3}{4}\right) + B + \frac{1}{2} u_2(1) + \frac{1}{2} u_3\left(\frac{1}{2}\right) \\
= &- \frac{1}{4} + B + \frac{1}{2} u_2(1) + \frac{1}{2} u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
Eu_c(x''') = &- |x''' - \hat{x}_i| + B + \frac{1}{2}\left[u_2(1) + (1-\rho) * 0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
&+ \frac{1}{2}\left[0 + (1-\rho) * 0 + \rho u_3\left(\frac{1}{2}\right)\right] \\
= &- \left(\frac{3}{4} - 0\right) + B + \frac{1}{2} u_2(1) + \rho u_3\left(\frac{1}{2}\right) \\
= &- \frac{3}{4} + B + \frac{1}{2} u_2(1) + \rho u_3\left(\frac{1}{2}\right)
\end{aligned}
$$

Note the expected utilities for the politician in group C are exactly the same as those for the extreme politician in group B with respect to her first, second, or third choice policy. I conclude the politician from group C never compromises to her second choice policy in the first period; therefore, all politicians in group C choose $x_1 = \frac{1}{2}$.

Knowing the actions of politicians in group C and D, I identify $\beta_1 = \frac{B+2}{3}$, where politicians with ideal points to the right of β_1 choose $x_1 = 1$. Politicians with ideal points between α_1 and β_1 choose $x_1 = \frac{1}{2}$.

Voter Strategies:

In equilibrium, the voter re-elects a politician who chooses $x_1 = \omega_1$ or $x_1 = \frac{1}{2}$. Given the strategies of the politicians derived above, I now show this course of action is optimal for the voter. Without loss of generality, assume the state of the world is revealed to be zero, $\omega_1 = 0$.

$$\begin{aligned} Eu_v(M|x_1 = 0, \omega_1 = 0) &= u_v(0|0) + (1-\rho)u_v(0|0) + \rho u_v(P) \\ &= -\frac{5}{2}\rho + \rho^2 \end{aligned}$$

$$\begin{aligned} Eu_v(M|x_1 = 1, \omega_1 = 0) &= u_v(1|0) + (1-\rho)u_v(P|0) + \rho u_v(1|1) \\ &= -\frac{3}{2} + \frac{3}{2}\rho - \rho^2 \end{aligned}$$

$$\begin{gathered} Eu_v\left(M|x_1 = \frac{1}{2}, \omega_1 = 0\right) = \frac{1}{\beta_1 - \alpha_1}\{ \\ (\alpha_{2,0} - \alpha_1)\left\{u_v(0|0) + (1-\rho)\left[\left(\frac{1}{4} - \alpha_1\right)u_v(0|0) + \left(\alpha_{2,0} - \frac{1}{4}\right)u_v\left(\frac{1}{2}|0\right)\right] + \rho u_v(P)\right\} \\ + (\beta_{2,0} - \alpha_{2,0})\left[u_v\left(\frac{1}{2}|0\right) + u_v(P)\right] \\ +(\beta_1 - \beta_{2,0})\left\{u_v(1|0) + (1-\rho)u_v(P) + \rho\left[\left(\frac{3}{4} - \beta_{2,0}\right)u_v\left(\frac{1}{2}|1\right) + \left(\beta_1 - \frac{3}{4}\right)u_v(1|1)\right]\right\}\ \} \\ = \frac{1}{\beta_1 - \beta_{2,0}}\{\ (\alpha_{2,0} - \alpha_1)\left[\rho(1-\rho)\alpha_1 - \frac{1}{2}\alpha_{2,0} + \frac{1}{4}\rho^2 - \frac{7}{8}\rho + \frac{1}{8}\right] + \alpha_{2,0} - \beta_{2,0} \\ (\beta_1 - \beta_{2,0})\left[\frac{1}{2}\rho\beta_{2,0}\rho^2\beta_1 + \frac{3}{4}\rho^2 + \frac{9}{8}\rho - \frac{3}{2}\right]\ \} \end{gathered}$$

$$Eu_v(N|\omega_1 = 0) =$$

$$\alpha'_{2,0}u_v(0|0) + \alpha'_{2,0}(1-\rho)\left[\frac{1}{4}u_v(0|0) + \left(\alpha'_{2,0} - \frac{1}{4}\right)u_v\left(\frac{1}{2}|0\right)\right] + \alpha'_{2,0}\rho u_v(1|1)$$

$$+(\beta'_{2,0} - \alpha'_{2,0})u_v\left(\frac{1}{2}|0\right) + (\beta'_{2,0} - \alpha'_{2,0})(1-\rho)u_v(P) + (\beta'_{2,0} - \alpha'_{2,0})\rho u_v(1|1)$$

$$+(1-\beta'_{2,0})u_v(1|0) + (1-\beta'_{2,0})(1-\rho)u_v(P) + (1-\beta'_{2,0})\rho u_v(1|1)$$

$$= \alpha'_{2,0}\left\{\frac{1}{4}\rho^2 - \frac{15}{8}\rho + \frac{9}{8} - \frac{1}{2}(1-\rho)\alpha'_{2,0}\right\} - \rho\beta'_{2,0} - \rho^2 + \frac{3}{2}\rho - 1$$

In equilibrium, the voter should re-elect the incumbent if the incumbent chooses $x_1 = \frac{1}{2}$ or $x_1 = \omega_1$.

$$Eu_v(M|x_1 = 0, \omega_1 = 0) \geq Eu_v(N|\omega_1 = 0)$$

$$-\frac{5}{2}\rho + \rho^2 \geq \alpha'_{2,0}\left\{\frac{1}{4}\rho^2 - \frac{15}{8}\rho + \frac{9}{8} - \frac{1}{2}(1-\rho)\alpha'_{2,0}\right\} - \rho\beta'_{2,0} - \rho^2 + \frac{3}{2}\rho - 1$$

As ρ approaches zero:

$$0 \geq B\left\{\frac{9}{8} - B\right\} - 1$$

The office benefit varies within the range $\left(\frac{1}{4}, \frac{3}{4}\right)$, so the right hand side of the inequality varies within the ranges $\left(-\frac{11}{16}, -\frac{9}{16}\right)$, the entirety of which is less than zero, so the inequality holds strictly. The voter re-elects a politician who matches the state of the world in the first period.

$$Eu_v\left(M|x_1 = \frac{1}{2}, \omega_1 = 0\right) \geq Eu_v(N|\omega_1 = 0)$$

As the expected utilities are quite long, I immediately take the limit of each when ρ approaches zero.

$$\frac{12B - 9}{16B - 8} \geq B\left\{\frac{9}{8} - B\right\} - 1$$

It has just been discussed that the right hand side of the inequality varies within $\left(-\frac{11}{16}, -\frac{9}{16}\right)$. The lower limit of the left hand side is $\frac{3}{2}$ which is is strictly greater than upper bound of the right hand side. Thus, the left hand side is always strictly greater than the right hand side and the inequality holds strictly. The voter prefers to elect a politician who choose $x_1 = \frac{1}{2}$ over a random challenger.

Finally, I show the voter would rather elect a random challenger than re-elect a politician who chose $x_1 = -\omega_1$.

$$\begin{aligned} &Eu_v(M|x_1 = 1, \omega_1 = 0) \leq Eu_v(N|\omega_1 = 0) \\ &\qquad -\frac{3}{2} + \frac{3}{2}\rho - \rho^2 \leq \alpha'_{2,0}\left\{\frac{1}{4}\rho^2 - \frac{15}{8}\rho + \frac{9}{8} - \frac{1}{2}(1-\rho)\alpha'_{2,0}\right\} - \rho\beta'_{2,0} - \rho^2 + \frac{3}{2}\rho - 1 \end{aligned}$$

As ρ approaches zero:

$$-\frac{3}{2} \leq B\left\{\frac{9}{8} - B\right\} - 1$$

The left hand side of the inequality is strictly less than the lower bound of the range of the right hand side of the inequality. Thus, the inequality holds strictly. The voter prefers to elect a random challenger over a politician who chooses the opposite state of the world in the first period.

I now formally summarize the strategies and beliefs of the actors in the first period.

$$\sigma_1^{*M} = \begin{cases} x_1 = 0 & \text{if } \hat{x}_i \in [0, \alpha_1] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha_1, \beta_1) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta_1, 1] \end{cases}$$

$$\mu^*_{\omega_1=0} = \frac{1}{2} \qquad \mu^*_{\omega_1=1} = \frac{1}{2}$$

$$\sigma_1^{*v} = \begin{cases} M & \text{if } x_1 = \omega_1 \\ M & \text{if } x_1 = \frac{1}{2} \\ N & \text{if } x_1 = -\omega_1 \end{cases}$$

$$\mu^*_{1,\hat{x}_i}(x_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{1,\hat{x}_i}(x_1 = \frac{1}{2}) \sim U(\alpha_1, \beta_1) \quad \mu^*_{1,\hat{x}_i}(x_1 = 1) \sim U[\beta_1, 1]$$

Now, putting the equilibrium strategies and beliefs from each period together, I have the entire equilibrium.

First Period:

$$\sigma_1^{*M} = \begin{cases} x_1 = 0 & \text{if } \hat{x}_i \in [0, \alpha_1] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha_1, \beta_1) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta_1, 1] \end{cases}$$

$$\mu^*_{\omega_1=0} = \frac{1}{2} \qquad \mu^*_{\omega_1=1} = \frac{1}{2}$$

$$\sigma_1^{*v} = \begin{cases} M & \text{if } x_1 = \omega_1 \\ M & \text{if } x_1 = \frac{1}{2} \\ N & \text{if } x_1 = -\omega_1 \end{cases}$$

$$\mu^*_{1,\hat{x}_i}(x_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{1,\hat{x}_i}(x_1 = \frac{1}{2}) \sim U(\alpha_1, \beta_1) \quad \mu^*_{1,\hat{x}_i}(x_1 = 1) \sim U[\beta_1, 1]$$

Second period:

When M is in office:

$$\sigma_2^{*M}(\omega_1) = \begin{cases} x_1 = 0 & \text{if } \hat{x}_i \in [0, \alpha_{2,\omega_1}] \\ x_1 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha_{2,\omega_1}, \beta_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1 - \rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(M, \omega_2 = \omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(M, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases}$$

Beliefs following equilibrium actions:

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 0, \omega_1 = 0) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 0, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 0, \omega_1 = 0) = 1$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 1, \omega_1 = 1) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 1, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 1, \omega_1 = 1) \sim U\left[\beta_1, 1\right]$$

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = \frac{1}{2}) \sim U[\alpha_1, \alpha_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = \frac{1}{2}\right) \sim U\left(\alpha_{2,\omega_1}, \beta_{2,\omega_1}\right)$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = \frac{1}{2}) \sim U\left[\beta_{2,\omega_1}, \beta_1\right]$$

Beliefs following off-the-path actions:

$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 0, \omega_1 = 1) \sim U[0, \alpha_1] \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 0, \omega_1 = 1\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 0, \omega_1 = 1) = 1$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 0|M, x_1 = 1, \omega_1 = 0) = 0 \quad \mu^*_{2,\hat{x}_i}\left(x_2 = \frac{1}{2}|M, x_1 = 1, \omega_1 = 0\right) = \frac{1}{2}$$
$$\mu^*_{2,\hat{x}_i}(x_2 = 1|M, x_1 = 1, \omega_1 = 0) \sim U\left[\beta_1, 1\right]$$

When N is in office:

$$\sigma_2^{*N}(\omega_1)\begin{cases} x_2 = 0 & \text{if } \hat{x}_i \in [0, \alpha'_{2,\omega_1}] \\ x_2 = \frac{1}{2} & \text{if } \hat{x}_i \in (\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \\ x_1 = 1 & \text{if } \hat{x}_i \in [\beta'_{2,\omega_1}, 1] \end{cases}$$

$$\mu^*_{\omega_2=\omega_1} = 1-\rho \qquad \mu^*_{\omega_2=-\omega_1} = \rho$$

$$\sigma_2^{*v}(N, \omega_2 = \omega_1) = \begin{cases} N & \text{i f } x_2 = \omega_2 \\ P & \text{if } x_2 = \frac{1}{2} \\ P & \text{if } x_2 = -\omega_2 \end{cases} \qquad \sigma_2^{*v}(N, \omega_2 = -\omega_1) = \begin{cases} M & \text{if } x_2 = \omega_2 \\ M & \text{if } x_2 = \frac{1}{2} \\ M & \text{if } x_2 = -\omega_2 \end{cases}$$

$$\mu^*_{2,\hat{x}_i}(x_2|N) \sim U[0, \alpha'_{2,\omega_1}] \quad \mu^*_{2,\hat{x}_i}(x_2|N) \sim U(\alpha'_{2,\omega_1}, \beta'_{2,\omega_1}) \quad \mu^*_{2,\hat{x}_i}(x_2 = 1|N) \sim U[\beta'_{2,\omega_1}, 1]$$

$$\alpha'_{2,0} = \frac{2(1-\rho)B + \rho}{2+2\rho} \qquad \beta'_{2,0} = \frac{3}{4}$$

$$\alpha'_{2,1} = 1 - \beta'_{2,0} \qquad \beta'_{2,1} = 1 - \alpha'_{2,0}$$

Third period:

$$\sigma_3^{*i} = \begin{cases} x_3 = 0 & \text{if } \hat{x}_i \in \left[0, \frac{1}{4}\right] \\ x_3 = \frac{1}{2} & \text{if } \hat{x}_i \in \left(\frac{1}{4}, \frac{3}{4}\right) \\ x_3 = 1 & \text{if } \hat{x}_i \in \left[\frac{3}{4}, 1\right] \end{cases}$$

$$\mu^*_{\omega_3=\omega_2} = 1-\rho \qquad \mu^*_{\omega_3=-\omega_2} = \rho$$